Proceedings of International Conference on "Recent Innovations in Engineering, Technology & Science for Sustainable Development (RIETSSD-2026)"
Career Point International Journal of Research (CPIJR)
©2022 CPIJR | Special Issue | ISSN : 2583-1895
Conference Paper | DOI: https://doi.org/10.5281/zenodo.20470815

A Type of Fractional Kinetic Equations Involving extended k-Generalized Mittag-Leffler Function

Dinesh Jain[1], Dr. Hemlata Saxena[2]

[1]Research Scholar, School of Basic & Applied Science, Career Point University, Kota (Rajasthan), India
[2]Professor School of Basic & Applied Science, Career Point University, Kota (Rajasthan), India
[1]dineshkotiya@gmail.com [2]saxenadrhemlata@gmail.com

Abstract:
This paper aims to develop a new generalized form of the fractional kinetic equation using extended k-generalized Mittag-Leffler function $E_{k,\alpha,\beta}^{\rho,\sigma,\gamma}(z,p)$. The solutions of fractional kinetic equations are discussed in terms of the Mittag-Leffler function and also evaluate the Laplace transform. The results established here are quite general in nature and capable of yielding both known and new results

Keywords: Fractional Kinetic Equations, Extended k-Generalized Mittag-Leffler Function, Fractional Calculus, Special Functions, Integral Transforms, Mathematical Modeling, Analytical Solutions

I. Introduction

Fractional-order calculus is a field of mathematics that extends the classical concepts of differentiation and integration to operators of arbitrary order, allowing the order to be any real or even complex number rather than being restricted to integers [1–8]. Although the Fractional-order calculusrepresents more than three centuries old issue [10, 11], significant attention to its theoretical research and real-world applications has emerged only in recent decades. The idea of nonintegral derivative was mentioned for the first time probably in a letter from Leibniz to L'Hôpital in 1695. Later on, the pioneering works related to Fractional-order calculus were elaborated by personalities such as Euler, Fourier, Abel, Liouville, or Riemann. The interested reader can find a more detailed historical background of theFractional-order calculus, in [7].

In recent years, however, this situation has improved significantly, and Fractional order Calculus has emerged as an effective mathematical framework for modelling phenomena such as fractal characteristics, long-term memory effects. Consequently, fractional-order calculus has already came in useful in engineering fields such as bioengineering, viscoelasticity, electronics, robotics, control theory, and signal processing. Several control applications are available, in [12–14].

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In view of the effectiveness and significant role of the kinetic equation in various astrophysical problems, the authors develop a more generalized form of the fractional kinetic equation.

Haubold and Mathai [15], introduced a fractional differential equation between the rate of change of the reaction, destruction rate, and the production rate, as given below.

$$\frac{dN}{d\tau} = -d(N_\tau) + s(N_\tau) \quad \dots (1.1)$$

where $N = N_\tau$ is the rate of reaction, $d = d(N_\tau)$ is the rate of destruction, $d = d(N_\tau)$ is the rate of production, and $N_\tau$is the function defined by $N_\tau(\tau^*) = N(\tau - \tau^*),\ \tau^* > 0$.

Aspecial case of (1.1) for spatial fluctuations and inhomogeneities in $N_\tau$where the quantities are neglected is the equation

$$\frac{dN}{d\tau} = -c_i N_i(\tau), \quad (c_i > 0) \quad \dots (1.2)$$

with the initial condition$N_i(\tau = 0) = N_0$ representing the number density of the species i at time $\tau = 0$ and$c_i > 0$. If we remove the index $i$and integrate the standard kinetic equation (1.2), is given by Haubold and Mathai [6] as follows:

$$N(\tau) - N_0 = -c^\upsilon {}_0D_t^{-1} N(\tau) \quad \dots (1.3)$$

where ${}_0D_t^{-1}$is the particular case of the Riemann–Liouville integral operator ${}_0D_t^{-\upsilon}$defined as

$${}_0D_t^{-\upsilon} f(\tau) = \frac{1}{\Gamma(\upsilon)} \int_0^\tau (\tau - u)^{\upsilon - 1} f(u)du, \quad \tau > 0, \upsilon > 0 \quad \dots (1.4)$$

They obtained the solution of (1.3) as

$$N(\tau) = N_0 \sum_{k=0}^{\infty} \frac{(-1)^k}{\Gamma(\upsilon k + 1)} (c\tau)^{\upsilon k} \quad \dots (1.5)$$

Saxena and Kalla [15] introduced the following fractional kinetic equation:

$$N(\tau) - N_0 f(\tau) = -c^\upsilon {}_0D_t^{-\upsilon} N(\tau), \quad R(\upsilon) > 0 \quad \dots (1.6)$$

where $N(\tau)$ denotes the number density of a given species at time τ, $N_0 = N(0)$is the number density of that species at time $\tau = 0$, $c$is a constant, and $f \in L(0, \infty)$Applying the Laplace transform to (1.6) (see [16]), we have

$$L(N(\tau); s) = N_0 \frac{F(p)}{1 + c^\upsilon s^{-\upsilon}} = N_0 \left(\sum_{n=0}^{\infty} (-c^\upsilon)^n s^{-\upsilon n} \right) F(s), \quad n \in N_0, \left|\frac{c}{s}\right| < 1 \quad \dots (1.7)$$

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The Laplace Transform of the operator defined in (1.3) is given by [2]

$$L[{}_0D_t^{-\upsilon} f(\tau); s] = s^{-\upsilon} F(s) \quad \text{... (1.8)}$$

$$F(s) = L(N(\tau); s) = \int_0^\infty e^{-s\tau} f(\tau) d\tau, \quad R(s) > 0 \quad \text{... (1.9)}$$

We further need some additional definitions and functions. Recently, Diazand Pariguan [19], have introduced the k-Pochhammer symbol defined as follows:

$$(\xi)_{n,k} = \xi(\xi + k)(\xi + 2k)(\xi + 3k) \dots\dots\dots (\xi + (n-1)k \quad \text{... (1.10)}$$

$$(1 - z)^{-\gamma} = \sum_{n=0}^{\infty} \frac{(\gamma)_n}{(n)!} z^n \quad \text{... (1.11)}$$

Where $\gamma \in C, \; n \in N$.

Proposition 1 If $\gamma \in C \; and \; k, s \in R$, given then the following identity is true: (see in [17])

$$\Gamma s(\xi) = \left(\frac{s}{k}\right)^{\frac{\xi}{s}-1} \Gamma k\left(\frac{k\xi}{s}\right) \text{ and in particular } \Gamma k(\xi) = k^{\frac{\xi}{k}-1} \Gamma\left(\frac{\xi}{k}\right) \quad \text{... (1.12)}$$

Proposition 2 If $\xi \in C$, and $k, s \in R$, and $n \epsilon N$ given then the following identity is true [18]:

$$(\xi)_{n\tau,s} = \left(\frac{s}{k}\right)^{n\tau} \left(\frac{k\xi}{s}\right)_{n\tau,k}, \text{ and in particular } (\xi)_{n\tau,k} = k^{n\tau} \left(\frac{\xi}{k}\right)_{n\tau} \quad \text{... (1.13)}$$

Proposition 3 For$> 0, x > 0, y > 0$, the following integral representation of k Beta function and its relation with k-Gamma function [25] holds true:

$$\mathrm{B_k}(x, y) = \frac{1}{k} \int_0^1 u^{\frac{x}{k}-1} (1-u)^{\frac{y}{k}-1} \, dt = \frac{\Gamma_k(x)\Gamma_k(y)}{\Gamma_k(x+y)} \quad \text{... (1.14)}$$

Proposition 4 For $k > 0, x > 0, y > 0, p > 0$, the following integral representation of $\mathrm{B_k}(x, y; p)$ holds true as an extension of the k- Beta function [26].

$$\mathrm{B_k}(x, y; p) = \frac{1}{k} \int_0^1 u^{\frac{x}{k}-1} (1-u)^{\frac{y}{k}-1} e^{-\frac{p^k}{kt(1-t)}} \, dt, \quad \text{... (1.15)}$$

Extended k-generalized Mittag-Leffler function:

An extension of various functions discussed above is provided by the definition of extended k-generalized Mittag-Leffler function, which is given as follows [24]:

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$$E_{k,\alpha,\beta}^{\rho,\sigma,\gamma}(x;p)=\sum_{n=0}^{\infty}\frac{B_k(\rho+n\sigma k,\gamma-\rho;p)}{B_k(\rho,\ \gamma-\rho)}\frac{(\gamma)_{n\sigma,k}}{\Gamma_k(\alpha n+\beta)}\frac{x^n}{n!} \quad ...(1.16)$$

Where $k>0;\ \rho,\alpha,\beta,\gamma,x\in C$ and $Re(\alpha)>0, Re(\beta)>0, Re(\rho)>0, \sigma\in(0,1)\cup N; p\geq 0$.

Special Cases:

(i) Taking $\sigma=1$ in (1.16), we have extended k-Mittag-Leffler function [21].

(ii)Taking $\sigma=k=1$ in (1.16), we have extended Mittag-Leffler function [20].

(iii) Considering $\sigma=k=1,\ p=0$ in (1.16), we have Mittag-Leffler function [22].

For $k>0$,integral representation of extended k-generalized Mittag-Leffler function is given as follows [24].

$$E_{k,\alpha,\beta}^{\rho,\sigma,\gamma}(x;p)=\frac{1}{k\,B_k(\rho,\gamma-\rho)}\int_0^1 t^{\frac{\rho}{k}-1}(1-t)^{\frac{\gamma-\rho}{k}-1}e^{-\frac{p^k}{kt(1-t)}}\,G E_{k,\alpha,\beta}^{\gamma,\sigma}(xt^{\sigma})\ dt \quad ...(1.17)$$

Suppose that $f(t)$ is a real (or complex) valued function of the (time) variable t > 0 and $(s)$is a real or complex parameter. The Laplace transform of the function $f(t)$ is defined by

$$F(s)=L\{f(t);s\}=\int_0^{\infty} e^{-st} f(t)dt\,, \qquad Re(s)>0 \quad ...(1.18)$$

The convolution of two functions $f(t)$ and $g(t)$, which are defined for $t>0$, plays an important role in a number of different physical applications. The Laplace convolution of the functions $f(t)$ and $g(t)$ is given by the following integral [23]:

$$(g*f)(t)=(f*g)(t)=\int_0^t f(u)\ g(t-u)\ du \quad ...(1.19)$$

Theorem 1.1 The Laplace Transform of the extended k-generalized Mittag-Leffler function is as follows:

Proof: Using the definition of Laplace transform (1.18).

$$L\left\{E_{k,\alpha,\beta}^{\rho,\sigma,\gamma}(x;p);s\right\}=L\left\{\int_0^{\infty} e^{-st}E_{k,\alpha,\beta}^{\rho,\sigma,\gamma}(x;p)dt\right\} \quad ...(1.20)$$

Integral representation of extended k-generalized Mittag-Leffler function is given as follows.

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$$E_{k,\alpha,\beta}^{\rho,\sigma,\gamma}(x;p) = \frac{1}{kB_k(\rho,\gamma-\rho)} \int_0^1 t^{\frac{\rho}{k}-1}(1-t)^{\frac{\gamma-\rho}{k}-1} e^{-\frac{p^k}{kt(1-t)}}\, dt \sum_{n=0}^{\infty} \frac{(\gamma)_{n\sigma,k}}{\Gamma_k(\alpha n+\beta)} \frac{(xt^\sigma)^n}{n!} \quad \dots(1.21)$$

$$L\left\{E_{k,\alpha,\beta}^{\rho,\sigma,\gamma}(x;p);s\right\} = \frac{B_k(\rho,\gamma-\rho;p)}{B_k(\rho,\gamma-\rho)} \int_0^\infty \sum_{n=0}^{\infty} e^{-st} \frac{(\gamma)_{n\sigma,k}}{\Gamma_k(\alpha n+\beta)} \frac{(xt^\sigma)^n}{n!}\, dt$$

$$= \frac{B_k(\rho,\gamma-\rho;p)}{B_k(\rho,\gamma-\rho)} \sum_{n=0}^{\infty} \frac{(\gamma)_{n\sigma,k}}{\Gamma_k(\alpha n+\beta)} \frac{x^n}{n!} \int_0^\infty e^{-st} t^{n\sigma}\, dt$$

$$= \sum_{n=0}^{\infty} \frac{B_k(\rho,\gamma-\rho;p)}{B_k(\rho,\gamma-\rho)} \frac{(\gamma)_{n\sigma,k}}{\Gamma_k(\alpha n+\beta)} \frac{x^n}{n!} \frac{\Gamma(n\sigma+1)}{s^{n\sigma+1}} \quad \dots(1.22)$$

2. Solution of generalized fractional kinetic equations

In this section, we investigated the solutions of the generalized fractional kinetic equations by using the extended k-generalized Mittag-Leffler function.

Theorem 2.1: If

$c > 0, \upsilon > 0, k > 0,\ \rho,\alpha,\beta,\gamma,x \in C$ and $Re(\alpha) > 0, Re(\beta) > 0, Re(\rho) > 0, \sigma \in (0,1) \cup N; p \geq 0$

then the equation

$$N(\tau) - N_0\, E_{k,\alpha,\beta}^{\rho,\sigma,\gamma}(x;p) = -c^\upsilon\, {}_0D_t^{-\upsilon} N(\tau), \quad R(\upsilon) > 0 \quad \dots(2.1)$$

has the following solution

$$N(t) = N_0 \sum_{n=0}^{\infty} \frac{B_k(\rho,\gamma-\rho;p)}{B_k(\rho,\gamma-\rho)} \frac{(\gamma)_{n\sigma,k}\Gamma(n\sigma+1)}{\Gamma_k(\alpha n+\beta)} \frac{(xt^\sigma)^n}{n!} E_{\upsilon,\sigma n+1}(-c^\upsilon t^\upsilon) \quad \dots(2.2)$$

Proof: Taking the Laplace transform on both sides of (2.1), we obtain

$$L\{N(\tau);s\} = N_0 L\left\{E_{k,\alpha,\beta}^{\rho,\sigma,\gamma}(x;p);s\right\} - c^\upsilon\, L\{{}_0D_t^{-\upsilon}N(\tau);s\}$$

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$$N(s) = N_0 \int_0^\infty e^{-st} \left\{ E_{k,\alpha,\beta}^{\rho,\sigma,\gamma}(x;p); s \right\} dt - c^\upsilon s^{-\upsilon} N(s) \quad \text{... (2.3)}$$

which can be written as

$$N(s)(1 + c^\upsilon s^{-\upsilon}) = N_0 \int_0^\infty e^{-st} \left\{ E_{k,\alpha,\beta}^{\rho,\sigma,\gamma}(x;p); s \right\} dt$$

$$N(s)(1 + c^\upsilon s^{-\upsilon}) = \frac{B_k(\rho,\gamma-\rho;p)}{B_k(\rho,\gamma-\rho)} N_0 \sum_{n=0}^{\infty} \frac{(\gamma)_{n\sigma,k}}{\Gamma_k(\alpha n+\beta)} \frac{x^n}{n!} \frac{\Gamma(n\sigma+1)}{s^{n\sigma+1}} \quad \text{... (2.4)}$$

which on simplification gives

$$N(s) = \frac{B_k(\rho,\gamma-\rho;p)}{kB_k(\rho,\gamma-\rho)} N_0 \sum_{n=0}^{\infty} \frac{(\gamma)_{n\sigma,k}\Gamma(n\sigma+1)}{\Gamma_k(\alpha n+\beta)} \frac{x^n}{n!} \left\{ s^{-(n\sigma+1)} \sum_{r=0}^{\infty} \left[-\left(\frac{s}{c}\right)^{-\upsilon} \right]^r \right\} \quad \text{... (2.5)}$$

Now taking inverse Laplace of equation (2.5) and by using the result given as follows:

$$L^{-1}\{s^{-\upsilon}; t\} = \frac{t^{\upsilon-1}}{\Gamma(\upsilon)} \quad , \quad R(\upsilon) > 0 \quad \text{... (2.6)}$$

We have

$$L^{-1}N(s) = N_0 \frac{B_k(\rho,\gamma-\rho;p)}{B_k(\rho,\gamma-\rho)} N_0 \sum_{n=0}^{\infty} \frac{(\gamma)_{n\sigma,k}\Gamma(n\sigma+1)}{\Gamma_k(\alpha n+\beta)} \frac{x^n}{n!} \left\{ \sum_{r=0}^{\infty} (-1)^r (c)^{\upsilon r} L^{-1}\left\{ s^{-(n\sigma+\upsilon r+1)} \right\} \right\}$$

after simplification, we have

$$N(t) = \frac{B_k(\rho,\gamma-\rho;p)}{B_k(\rho,\gamma-\rho)} N_0 \sum_{n=0}^{\infty} \frac{(\gamma)_{n\sigma,k}\Gamma(n\sigma+1)}{\Gamma_k(\alpha n+\beta)} \frac{x^n}{n!} \left\{ \sum_{r=0}^{\infty} (c)^{\upsilon r} \frac{(-1)^r t^{\sigma n+\upsilon r}}{\Gamma(n\sigma+\upsilon r+1)} \right\} \quad \text{... (2.7)}$$

which can be written as

$$N(t) = \frac{B_k(\rho,\gamma-\rho;p)}{B_k(\rho,\gamma-\rho)} N_0 \sum_{n=0}^{\infty} \frac{(\gamma)_{n\sigma,k}\Gamma(n\sigma+1)}{\Gamma_k(\alpha n+\beta)} \frac{(xt^\sigma)^n}{n!} \left\{ \sum_{r=0}^{\infty} \frac{(-c^\upsilon t^\upsilon)^r}{\Gamma(n\sigma+\upsilon r+1)} \right\} \quad \text{... (2.8)}$$

the above equation (2.8) gives the required result (2.2).

Theorem 2.2: If

$c > 0,\ \delta > 0, \upsilon > 0,\ k > 0,\ \rho, \alpha, \beta, \gamma, x \in C$ and $Re(\alpha) > 0, Re(\beta) > 0,\ Re(\rho) > 0, \sigma \in (0,1) \cup N; p \geq 0$

then the equation

$$N(\tau) - N_0 t^{\delta-1}\, E_{k,\alpha,\beta}^{\rho,\sigma,\gamma}(x,p) = -\left\{\sum_{j=1}^{\infty} \binom{\mu}{j} (c^{\upsilon})^j\, {}_0D_t^{-\upsilon j}\right\} N(\tau) \qquad \ldots (2.9)$$

has the following solution

$$N(t) = t^{\delta-1} N_0 \sum_{n=0}^{\infty} \frac{B_k(\rho, \gamma-\rho; p)}{B_k(\rho, \gamma-\rho)} \frac{(\gamma)_{n\sigma,k}\, \Gamma(n\sigma+\delta)}{\Gamma_k(\alpha n+\beta)} \frac{(xt^{\sigma})^n}{n!} E_{\upsilon, n\sigma+\delta}^{\mu}(-c^{\upsilon}t^{\upsilon}) \qquad \ldots (2.10)$$

Proof: Taking the Laplace transform on both sides of (2.9), now using (1.17), we obtain

$$N(s) = N_0 \int_0^{\infty} e^{-st} \left\{t^{\delta-1}\, E_{k,\alpha,\beta}^{\rho,\sigma,\gamma}(x;\ p); s\right\} dt - c^{\upsilon} s^{-\upsilon} N(s) \qquad \ldots (2.11)$$

which can be written as

$$N(s)(1 + c^{\upsilon}s^{-\upsilon}) = \frac{B_k(\rho, \gamma-\rho; p)}{B_k(\rho, \gamma-\rho)} N_0 \sum_{n=0}^{\infty} \frac{(\gamma)_{n\sigma,k}}{\Gamma_k(\alpha n+\beta)} \frac{\Gamma(n\sigma+\delta)}{s^{n\sigma+\delta}} \frac{x^n}{n!} \qquad \ldots (2.12)$$

which on simplification gives

$$N(s) = \frac{B_k(\rho, \gamma-\rho; p)}{kB_k(\rho, \gamma-\rho)} N_0 \sum_{n=0}^{\infty} \frac{(\gamma)_{n\sigma,k}\Gamma(n\sigma+\delta)}{\Gamma_k(\alpha n+\beta)} \frac{x^n}{n!} \left\{s^{-(n\sigma+\delta)} \sum_{r=0}^{\infty} \left[-\left(\frac{s}{c}\right)^{-\upsilon}\right]^r\right\} \qquad \ldots (2.13)$$

Now taking inverse Laplace of equation (2.13) and by using the result (2.6), we have

$$L^{-1}N(s) = N_0 \frac{B_k(\rho, \gamma-\rho; p)}{B_k(\rho, \gamma-\rho)} N_0 \sum_{n=0}^{\infty} \frac{(\gamma)_{n\sigma,k}\Gamma(n\sigma+\delta)}{\Gamma_k(\alpha n+\beta)} \frac{x^n}{n!} \left\{\sum_{r=0}^{\infty} (-1)^r (c)^{\upsilon r} L^{-1}\left\{s^{-(\sigma n+\upsilon r+\delta)}\right\}\right\}$$

after simplification, we can be written as

$$N(t) = \frac{B_k(\rho, \gamma-\rho; p)}{B_k(\rho, \gamma-\rho)} N_0 \sum_{n=0}^{\infty} \frac{(\gamma)_{n\sigma,k}\Gamma(n\sigma+\delta)}{\Gamma_k(\alpha n+\beta)} \frac{(xt^{\sigma})^n}{n!} t^{\delta-1} \left\{\sum_{r=0}^{\infty} \frac{(-c^{\upsilon}t^{\upsilon})^r}{\Gamma(n\sigma+\upsilon r+\delta)}\right\} \qquad \ldots (2.14)$$

the above equation (2.14) gives the required result (2.10).

Theorem 2.3: If

$c > 0,\ \upsilon > 0,\ k > 0,\ \rho, \alpha, \beta, \gamma, x \in C$ and $Re(\alpha) > 0, Re(\beta) > 0, Re(\rho) > 0,\ \sigma \in (0,1) \cup N; p \geq 0$

then the equation

$$N(\tau) - N_0\ E_{k,\alpha,\beta}^{\rho,\sigma,\gamma}(x;p) = -c^{\upsilon}\ {}_0D_t^{-\upsilon} N(\tau), \quad R(\upsilon) > 0 \qquad \text{... (2.15)}$$

has the following solution

$$N(t) = N_0 \sum_{n=0}^{\infty} \frac{B_k(\rho, \gamma - \rho; p)}{B_k(\rho, \gamma - \rho)} \frac{(\gamma)_{n,k}\, \Gamma(n+1)}{\Gamma_k(\alpha n + \beta)} \frac{(xt)^n}{n!} E_{\upsilon,n+1}(-c^{\upsilon} t^{\upsilon}) \qquad \text{... (2.16)}$$

In the same process of analysis as in Theorems 2.1 and 2.2, we can find solutions of the generalized fractional kinetic equations involving the extended k-generalized Mittag-Leffler function (1.16). Proof of Theorem 2.3 is similar to Theorem 2.1, if we take $\sigma = 1$.

Special Cases:

By setting different values of the parameters, certain interesting results are obtained as follows:

On setting $\sigma = k = 1$, results in Theorem 1.1, reduce to the following form:

Corollary 1**:** If

$c > 0,\ \upsilon > 0,\ k > 0,\ \rho, \alpha, \beta, \gamma, x \in C$ and $Re(\alpha) > 0,\ Re(\beta) > 0,\ Re(\rho) > 0,\ \sigma \in (0,1) \cup N; p \geq 0$

then the equation

$$N(\tau) - N_0\ E_{k,\alpha,\beta}^{\rho,\sigma,\gamma}(x;p) = -c^{\upsilon}\ {}_0D_t^{-\upsilon} N(\tau), \quad R(\upsilon) > 0 \qquad \text{... (2.17)}$$

has the following solution

$$N(t) = N_0 \sum_{n=0}^{\infty} \frac{B(\rho, \gamma - \rho; p)}{B(\rho, \gamma - \rho)} \frac{(\gamma)_n\, \Gamma(n+1)}{\Gamma(\alpha n + \beta)} \frac{(xt)^n}{n!} E_{\upsilon,n+1}(-c^{\upsilon} t^{\upsilon}) \qquad \text{... (2.18)}$$

On setting $\sigma = k = 1,\ p = 0$, results in Theorem 2.2, reduce to the following form:

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Corollary 2**:** If

$c > 0,\ \ \delta > 0,\ \ \upsilon > 0,\ \ k > 0,\ \ \rho, \alpha, \beta, \gamma, x \in C$ and $Re(\alpha) > 0,\ \ \ Re(\beta) > 0,\ Re(\rho) > 0, \sigma \in (0,1) \cup N; p = 0$

then the equation

$$N(\tau) - N_0 t^{\delta-1} E_{k,\alpha,\beta}^{\rho,\sigma,\gamma}(x,p) = -\left\{\sum_{j=1}^{\infty} \binom{\mu}{j} (c^{\upsilon})^j\, {}_0D_t^{-\upsilon j}\right\} N(\tau) \qquad \text{... (2.19)}$$

has the following solution

$$N(t) = t^{\delta-1} N_0 \sum_{n=0}^{\infty} \frac{(\gamma)_n\, \Gamma(n+\delta)}{\Gamma\,(\alpha n+\beta)} \frac{(xt\,)^n}{n!} E_{\upsilon,n+\delta}^{\mu}(-c^{\upsilon} t^{\upsilon}) \qquad \text{... (2.20)}$$

3. **Conclusion**

Fractional kinetic equations involving the extended k-generalized Mittag–Leffler function are investigated. The derived solutions are expressed in terms of the Mittag–Leffler function. By assigning different values to the parameters, several interesting special cases are obtained. Moreover, due to the close relationship between the extended k-generalized Mittag–Leffler function and other special functions, further generalized forms of fractional kinetic equations can be developed, which may be very useful in various areas of basic sciences and engineering.

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| Special Issue | ISSN : 2583-1895
Conference Paper | DOI: https://doi.org/10.5281/zenodo.20470815

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