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Secure Quantum Teleportation of Squeezed Coherent States in a Noisy Gaussian Environment

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Abstract:

We investigate secure quantum teleportation (SQT) of squeezed coherent states in the presence of a noisy Gaussian environment. Unlike most previous studies that consider coherent states as the input to be teleported, we take squeezed coherent states as the information carriers and analyse how the squeezing of the input state influences the performance and security of the teleportation protocol. The quantum channel is modelled by a two-mode squeezed vacuum state shared between Alice and Bob, which interacts with a common squeezed thermal reservoir. The security of teleportation is ensured by simultaneously satisfying two conditions: a teleportation fidelity exceeding the classical limit $F>2/3$ and the existence of two-way quantum steering of the resource state. Using the covariance matrix formalism and the Lindblad master equation for open quantum systems, we study the time evolution of teleportation fidelity and Gaussian steering under the combined effects of dissipation, temperature, and environmental squeezing. The results show that the squeezing of the input coherent state plays a significant role in modifying the robustness of secure quantum teleportation against environmental noise. In particular, for suitable parameter regimes, input-state squeezing can enhance the tolerance of the protocol to thermal fluctuations and de-coherence, extending the temporal range in which SQT is achievable. These findings highlight the importance of the choice of input states in continuous-variable quantum communication protocols and provide new insights for optimizing secure quantum teleportation in realistic noisy environments.

Keywords: Teleportation**,** Entangled states, teleportation fidelity, Gaussian state, temporal range, SQT

1. Introduction: Quantum teleportation is a fundamental protocol in quantum information science that enables the transfer of an unknown quantum state from one location to another using shared entanglement and classical communication [1]. Since its original formulation and its extension to continuous-variable (CV) systems [2, 3], quantum teleportation has become a key component of quantum communication networks and the quantum internet [4, 5]. In CV platforms, Gaussian states and Gaussian operations play a central role due to their experimental accessibility and robustness [6,7].In realistic scenarios, quantum systems inevitably interact with their surrounding environment, which leads to dissipation and de-coherence [8,9]. These effects degrade entanglement and other quantum correlations, thereby reducing the efficiency and reliability of quantum teleportation [10]. For this reason, the study of teleportation in open quantum systems has attracted considerable attention, with various noisy channels such as thermal, squeezed, and correlated reservoirs being extensively investigated [11–13]. Secure quantum teleportation (SQT) is achieved when two conditions are simultaneously fulfilled: the teleportation fidelity must exceed the classical limit $F > 2/3$ [14,15], and the shared resource state must exhibit two-way quantum steering [16,17]. Quantum steering originates from the Einstein–Podolsky–Rosen paradox [18,19] and plays a crucial role in quantum information protocols including secure teleportation and quantum key distribution [20,21]. Most previous studies on SQT in continuous-variable systems have focused on coherent-state teleportation [3,22,23]. More general Gaussian states, such as squeezed and squeezed coherent states, naturally arise in quantum optics and offer enhanced capabilities [6,24,25]. In particular, squeezed coherent states combine displacement and quadrature squeezing, making them promising candidates as carriers of quantum information [26–28].

In this paper, we investigate secure quantum teleportation of squeezed coherent states in a noisy Gaussian environment. We consider a two-mode squeezed vacuum state as the shared quantum resource [29,30], interacting with a common squeezed thermal reservoir [11,13]. The dynamics are described using the Lindblad master equation [31, 8], and the covariance

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matrix formalism [6,32]. Security is quantified by teleportation fidelity [10, 15] and two-way Gaussian quantum steering [17,33].

The structure of the paper is as follows. The essential components of SQT of a coherent state are introduced in Section 2. The impact of the noisy environment on SQT is described in Section 3. The results are described in Section 4, and the concluding remarks are included in the last section.

2. Secure Quantum Teleportation:

Secure quantum teleportation (SQT) is achieved when both high teleportation fidelity and genuine quantum security are guaranteed. In continuous-variable systems, this requires the simultaneous fulfilment of two conditions:

(i) the teleportation fidelity must be larger than the classical limit F>2/3 [14,15]

(ii) the shared resource state between Alice and Bob must exhibit two-way quantum steering [16,17,21].

Here the bound F>2/3 represents the optimal fidelity achievable by any classical measure-and-prepare strategy [14,15], while two-way quantum steering ensures that both parties can non-locally influence each other's states, certifying the genuinely quantum and secure nature of the teleportation process [16,17,21].

Secure quantum teleportation is achieved when

$$F > \frac{2}{3}, S_{A\to B} > 0, S_{B\to A} > 0$$

For convenience, we define the quantity

$$L = \min\left\{S_{A\to B}, S_{B\to A}, F - \frac{2}{3}\right\}$$

The teleportation protocol is secure if and only if

$$L > 0$$

2.1 Teleportation fidelity for squeezed coherent input states:

The fidelity between an input state ρ_{in} and an output state ρ_{out} is defined as [10,15,34]

$$F(\rho_{\text{in}}, \rho_{\text{out}}) = \left[\text{Tr}\left(\sqrt{\sqrt{\rho_{\text{out}}}\,\rho_{\text{in}}\sqrt{\rho_{\text{out}}}}\right)\right]^2$$

Assuming ideal displacement correction in the teleportation protocol,

$$X_{out} = X_{in},$$

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the fidelity reduces to

$$F = \frac{1}{\sqrt{\Delta}}, \Delta = \det(\sigma_{in} + \sigma_{out})$$

Where σ_{in} and σ_{out} are the covariance matrices of the input and output states, respectively.

In the Braunstein–Kimble protocol, the output covariance matrix of the teleported state is obtained from the input state and the shared two-mode Gaussian resource through a linear transformation that incorporates the Bell measurement and classical feed-forward operations [3,6].

$$\sigma_{\text{out}} = \sigma_{\text{in}} + \Sigma,$$

Where Σ depends only on the covariance matrix of the shared bipartite resource state

$$\sigma_{AB}(t) = \begin{pmatrix} A(t) & C(t) \\ C^T(t) & B(t) \end{pmatrix}.$$

The matrix Σ has the form

$$\Sigma = \begin{pmatrix} X & Z \\ Z & Y \end{pmatrix},$$

With

$$X = A_{11} + B_{11} - 2C_{11}$$
$$Y = A_{22} + B_{22} + 2C_{22}$$
$$Z = A_{12} - B_{12} + C_{12} - C_{21}$$

Hence, the teleportation fidelity becomes

$$F = \frac{1}{\sqrt{\det(2\sigma_{in} + \Sigma)}}$$

In this work, the input state is chosen as a squeezed coherent state

$$|\alpha, \zeta\rangle = D(\alpha)S(\zeta)|0\rangle$$

Where $D(\alpha)$ and $S(\zeta)$ are the displacement and squeezing operators, respectively, and $\zeta = se^{i\phi}$. For simplicity, we take $\phi = 0$

The covariance matrix of a squeezed coherent state is

$$\sigma_{in} = \frac{1}{2}\begin{pmatrix} e^{-2s} & 0 \\ 0 & e^{2s} \end{pmatrix},$$

which reduces to the coherent-state form $\sigma_{\text{in}} = \frac{1}{2} I_2$ for $s = 0$.

Therefore, the teleportation fidelity for squeezed coherent input states reads

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$$F = \frac{1}{\sqrt{\det\left(\begin{pmatrix} e^{-2s} & 0 \\ 0 & e^{2s} \end{pmatrix} + \Sigma\right)}}$$

Where Σ is the added noise matrix of the teleportation channel. This expression explicitly shows how the squeezing parameters of the input state modifies the teleportation fidelity [10,35,36]. This formula follows from the general expression of the fidelity between two single–mode Gaussian states in terms of their covariance matrices and first moments, and its application to continuous-variable teleportation with Gaussian resources [6,10,35–37].

2.2 The two-way quantum steering:

For a bipartite Gaussian state with covariance matrix

$$\sigma_{AB} = \begin{pmatrix} A & C \\ C^T & B \end{pmatrix}$$

The Gaussian steering from Alice to Bob is quantified by

$$S_{A\to B} = \max\left\{0, \frac{1}{2}\ln\left(\frac{\det A}{4\det\sigma_{AB}}\right)\right\}$$

And the steering from Bob to Alice by

$$S_{B\to A} = \max\left\{0, \frac{1}{2}\ln\left(\frac{\det B}{4\det\sigma_{AB}}\right)\right\}.$$

Two-way steering exists if and only if

$$S_{A\to B} > 0, S_{B\to A} > 0$$

This definition and quantification of Gaussian quantum steering were introduced in the seminal work by Kogias et al. and are now standard in continuous-variable quantum information [17]. The operational role of two-way steering in secure quantum teleportation and related protocols has been further emphasized [21, 33].

3. The Effect of Noisy Environment on SQT:

In realistic physical implementations, the bipartite system shared by Alice and Bob inevitably interacts with its surrounding environment. Such interactions give rise to dissipation and de-coherence, which degrade quantum correlations and consequently affect the performance and security of quantum teleportation [8,9,11]. In this work, we model the environment as a squeezed thermal reservoir and study its influence on the dynamics of teleportation fidelity and quantum steering, and hence on the conditions for secure quantum teleportation

[11,13,38,39].We consider an open quantum system composed of two uncoupled bosonic modes interacting with a common squeezed thermal bath. Within the Markovian approximation, the time evolution of the density operator ρ of the system is described by the Lindblad master equation [31,8,38].

$$\frac{\partial \rho}{\partial t} = \sum_{i=1}^{2} \frac{\gamma}{2}\left[(N+1)\left(2a_i\rho a_i^\dagger - a_i^\dagger a_i\rho - \rho a_i^\dagger a_i\right) + N\left(2a_i^\dagger \rho a_i - a_i a_i^\dagger \rho - \rho a_i a_i^\dagger\right)\right]$$
$$- \sum_{\substack{i,j=1 \\ i\neq j}}^{2} \frac{\gamma}{2}\left[M\left(2a_i^\dagger \rho a_j^\dagger - a_i^\dagger a_j^\dagger \rho - \rho a_i^\dagger a_j^\dagger\right) + M^*\left(2a_i\rho a_j - a_i a_j \rho - \rho a_i a_j\right)\right].$$

The parameters satisfy

$$|M|^2 \leq N(N+1)$$

The asymptotic covariance matrix is [40]

$$\sigma(\infty) = \begin{pmatrix} \left(N+\frac{1}{2}\right) I_2 & M\sigma_z \\ M\sigma_z & \left(N+\frac{1}{2}\right) I_2 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The bath parameters are

$$N = n_{th}(\cosh^2 R + \sinh^2 R) + \sinh^2 R,$$
$$M = -(2n_{th}+1)\cosh R \sinh R,$$

with

$$n_{th} = \frac{1}{2}\left(\coth \frac{1}{2T} - 1\right)$$

The covariance matrix evolves as

$$\sigma(t) = \Gamma\sigma(0) + (I_4 - \Gamma)\sigma(\infty), \Gamma = e^{-\gamma t} I_4$$

The initial resource state is taken as a two-mode squeezed vacuum [40]

$$\sigma(0) = \frac{1}{2}\begin{pmatrix} \cosh 2r & 0 & \sinh 2r & 0 \\ 0 & \cosh 2r & 0 & -\sinh 2r \\ \sinh 2r & 0 & \cosh 2r & 0 \\ 0 & -\sinh 2r & 0 & \cosh 2r \end{pmatrix}$$

The effect of the noisy environment on secure quantum teleportation is quantified by

$$L(t) = \min\left\{S_{A\to B}(t), S_{B\to A}(t), F(t) - \frac{2}{3}\right\}$$

The condition $L(t) > 0$ determines the temporal regions in which secure teleportation of squeezed coherent states is achievable.

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4. Results:

The results obtained from this study are shown below respectively. Figure 1 present three-dimensional surface plots of the teleportation fidelity F (t,r) as functions of time and squeezing. The three-dimensional surface of the teleportation fidelity F(t,r) illustrates the combined influence of environmental de-coherence and squeezing on the quality of teleportation. For a fixed squeezing parameter r, the fidelity decreases monotonically with time, reflecting the progressive loss of quantum coherence induced by the noisy Gaussian environment. Conversely, for a fixed interaction time t, the fidelity increases with the squeezing parameter, demonstrating that stronger squeezing enhances the robustness of squeezed coherent states against de-coherence. The region where F(t,r)>2/3 defines the domain of secure quantum teleportation, which expands significantly with increasing squeezing. This confirms that squeezing plays a crucial role in extending the temporal window and improving the reliability of secure quantum teleportation in noisy Gaussian channels.

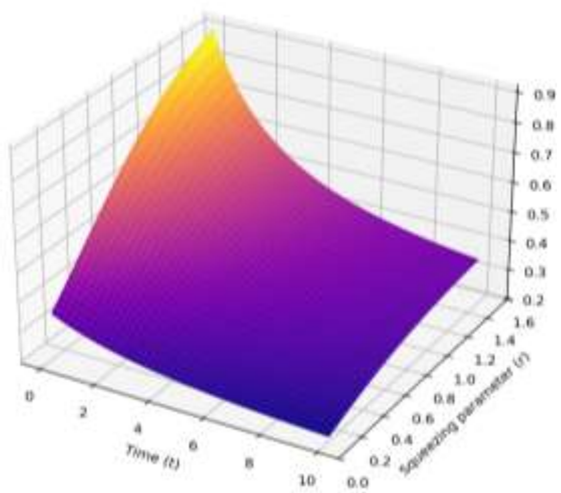

Figure: 1 3D teleportation fidelity for squeezed coherent states

Figures 2 (a) and (b) show the three-dimensional surfaces of the steering parameters $S_{A\rightarrow B}$ (t,r) and $S_{B\rightarrow A}$ (t,r) respectively. In both cases, steering increases with the squeezing parameter and decreases with time due to environmental de-coherence. However, $S_{A\rightarrow B}$ (t,r) is consistently larger than $S_{B\rightarrow A}$ (t,r), revealing the intrinsic asymmetry of quantum steering. This asymmetry implies that Alice has a stronger ability to influence Bob's state than Bob

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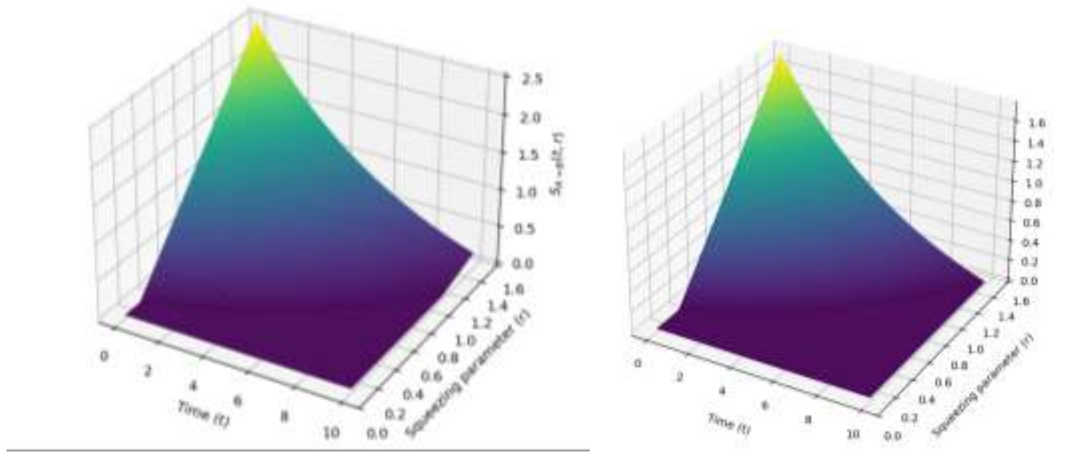
has to influence Alice's state, which is a crucial feature for secure quantum teleportation. The enhancement of steering with squeezing demonstrates the key role of squeezed coherent states in strengthening directional quantum correlations and improving the robustness of the teleportation protocol against Gaussian noise.



(a) (b)

Fig. 2: (a) , (b) three-dimensional surfaces of $S_{A\to B}$ (t,r) and $S_{B\to A}$ (t,r) respectively.

Fig. 3: illustrates the asymmetry parameter ΔS (t,r)=$S_{A\to B}$ (t,r)−$S_{B\to A}$ (t,r) as a function of time and squeezing. The positive values of ΔS\Delta SΔS over a wide parameter range demonstrate the existence of asymmetric quantum steering, with Alice-to-Bob steering being stronger than Bob-to-Alice steering. The asymmetry increases with the squeezing parameter, highlighting the crucial role of squeezing in enhancing directional quantum correlations. However, as time increases, environmental noise suppresses the asymmetry, eventually driving the system into a non-steerable classical regime. This behaviour confirms that squeezing enhances both the strength and robustness of secure quantum teleportation in noisy Gaussian environments.

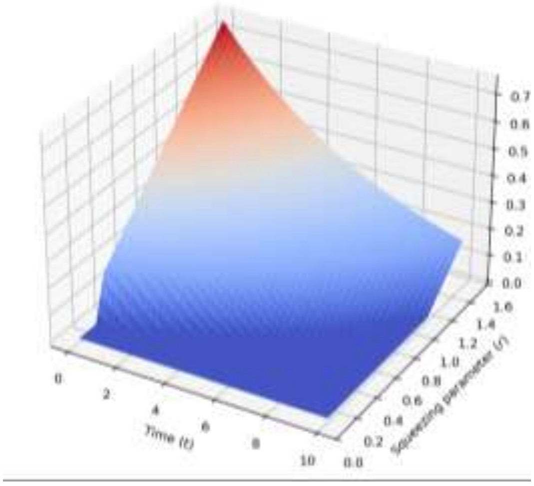


Figure: 3 Asymmetry surface: $S_{AOB}(t,r)$ - $S_{BOA}(t,r)$

Fig. 4: show the influence of Gaussian noise on teleportation fidelity. Increasing the de-coherence rate γ accelerates the decay of fidelity, indicating the destructive role of environmental noise. However, for small values of γ, non-classical correlations survive for longer interaction times, ensuring secure quantum teleportation.

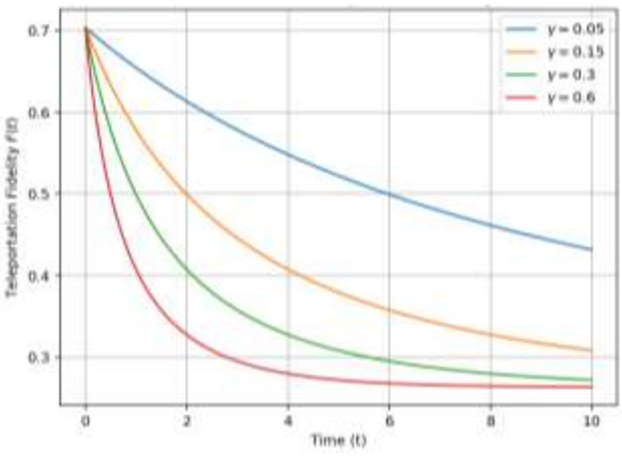


Figure: 4 Effect of Noise in TF

Figure 5 (a) illustrates the influence of Gaussian noise on the quantum steering parameter $S_{A\to B}$ (t). As the de-coherence rate γ increases, steering decays faster and vanishes at earlier times, demonstrating the detrimental effect of environmental noise on directional quantum correlations and the security of teleportation.(b) shows the noise dependence of the steering parameter $S_{B\to A}$ (t). Compared to $S_{A\to B}$ (t) , $S_{B\to A}$ (t) decay is faster and the maximum steering is smaller, confirming the asymmetric nature of quantum steering in a noisy Gaussian environment.

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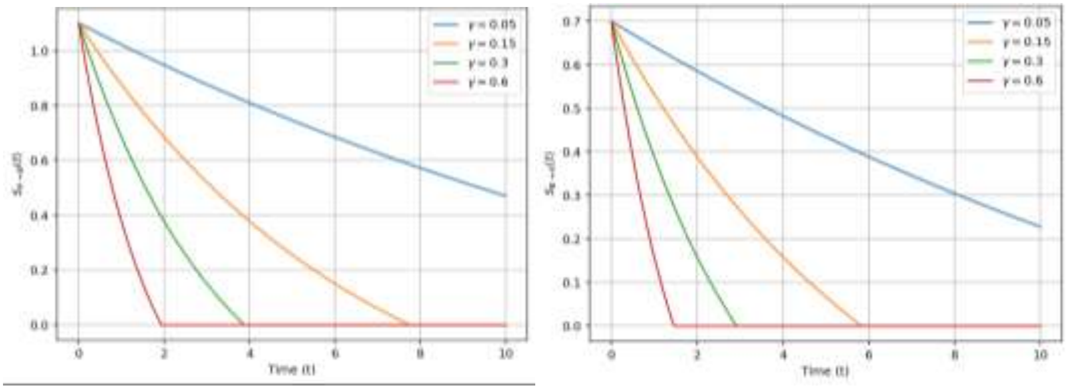

(a) (b)

Figure: 5 the effect of noise on quantum steering

5. Conclusion: In this work, we have investigated the secure quantum teleportation of squeezed coherent states in a noisy Gaussian environment by analysing the combined behaviour of teleportation fidelity, quantum steering, and steering asymmetry. Our approach provides a comprehensive and continuous description of security in continuous-variable quantum teleportation through three-dimensional surface representations of the relevant physical quantities. The results clearly demonstrate that the squeezing parameter of the input state plays a fundamental role in enhancing the performance and robustness of the teleportation protocol. An increase in squeezing leads to a significant improvement in the teleportation fidelity and enlarges the temporal window over which the fidelity remains above the classical threshold (F>2/3), which is required for secure teleportation. This establishes squeezed coherent states as more powerful and resilient information carriers compared to ordinary coherent states in noisy quantum channels.

Furthermore, our analysis of quantum steering reveals that both $S_{A\to B}$ (t,r) and $S_{B\to A}$ (t,r) increase with squeezing and decay with time due to environmental de-coherence. The consistent inequality

$S_{A\to B}$ (t,r) > $S_{B\to A}$ (t,r) confirms the intrinsic asymmetry of quantum steering in the system. This directional imbalance is a crucial resource for secure quantum communication, as it provides a natural form of one-way quantum control that cannot be reproduced by classical correlations.

Overall, our results identify three distinct operational regimes:

(i) a high-security quantum regime characterized by large squeezing and short interaction times, where fidelity, steering, and asymmetry are all significant;

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(ii) an intermediate regime where quantum features survive but are weakened; and

(iii) a classical regime at long times where fidelity drops, steering vanishes, and asymmetry disappears.

In conclusion, this study establishes squeezed coherent states as highly advantageous resources for secure quantum teleportation in noisy Gaussian environments. By simultaneously enhancing teleportation fidelity, strengthening quantum steering, and amplifying steering asymmetry, squeezing emerges as a key physical parameter for extending the duration, reliability, and directional security of quantum communication protocols. These findings provide important guidance for the design of practical continuous-variable quantum networks and contribute to a deeper understanding of the interplay between non-classical resources and environmental noise in secure quantum information processing.

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