

## **Quantum Theory and Quantum Entanglement: A Review of Quantum Correlations**

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### **Abstract:**

The most common type of quantum correlations is quantum entanglement, which plays an indisputable vital role in a number of quantum information theory tasks, including quantum teleportation, quantum dense coding, and quantum cryptography. However, current findings indicate that different Entanglement is not necessary for applications in quantum information theory, and their performance can be described by a novel kind of quantum correlations that transcends entanglement. The most well-liked candidate for such widespread quantum connections is quantum discord, which Zurek first proposed more than ten years ago. We provide an overview of this current line of inquiry in this work. Following a brief overview of the fundamental ideas of quantum theory and entanglement, we introduce quantum discord and generic quantum correlations and go over three applications that are predicated on this novel kind of correlation: transmission of correlations, entanglement dispersion, and remote state preparation. We also provide an outlook for future research in this area.

**Keywords:** Quantum information , quantum correlations, quantum teleportation, quantum cryptography, entanglement dispersion

### **1. INTRODUCTION**

Since the beginning of quantum theory, scientists have been captivated with quantum entanglement [1],[2]. Entangled quantum systems can exhibit peculiar behaviours that seem to go against "our common sense notions of how the world works" [3]. In a groundbreaking study, Einstein, Podolsky, and Rosen initially noted this and came to the conclusion that there must be gaps in the quantum theory [4]. But roughly thirty years after Einstein's criticism, Bell suggested an experiment that sought to differentiate between Einstein's reasoning and the



predictions of quantum theory [5]. Clauser, Horne, Shimony, and Holt developed an inequality that is now known as the CHSH inequality based on Bell's concepts [6]. According to Einstein et al., nature ought to respect the CHSH inequality, and the fact that it can be violated in quantum theory demonstrates the incompleteness of quantum mechanics. Freedman and Clauser were able to test the CHSH inequality empirically shortly after its discovery because of its simplicity [7].

The findings supported the quantum mechanical explanation of nature by demonstrating a violation of the CHSH inequality, which invalidated Einstein's claims. Aspect et al. conducted three trials later in 1981/82 [8]-[10], corroborating Freedman and Clauser's findings. Since then, a number of studies have showed a breach of the CHSH inequality, however there will be some gaps [11]. Werner expanded the concept of entanglement to include all mixed quantum states in 1989, which is when the formal definition of entanglement as we understand it today originated [12]. The theory of entanglement, which examines the characteristics and consequences of entanglement as well as its application to basic tasks like quantum cryptography [13], quantum dense coding [14], and teleportation [15], can be seen as having its roots in Werner's work. The Horodecki family also made several significant contributions to the theory of entanglement, including the finding of bound entanglement is one example [16]. Although they can't be utilized to extract any pure entangled state, bound entangled states require some entanglement to be formed.

Provides a thorough analysis of this subject. Entanglement's function in quantum algorithms is still hotly contested. This is because Jozsa and Linden's findings shown that entanglement is necessary for a quantum computer working in a pure state to achieve an exponential speedup over classical processing. [17], [18]. There is compelling evidence for the existence of a quantum computer, even though its exponential speedup has not yet been conclusively demonstrated. Shor's prime factorization algorithm, which was presented in [19], is among the most well-known examples pointing in this direction. On a quantum computer, the procedure may determine the prime factors for any product of two primes; the computing time increases polynomially with the number of input bits. Compared to the most well-known classical approach, which shows an exponential increase in running time, this is substantially faster. One could be tempted to view entanglement as the primary resource for quantum computation because it is present in Shor's algorithm. This is true for pure state quantum



computation, but when mixed state quantum computation is taken into account, things get more complicated. A popular example for mixed state quantum computation has been presented by Knill and Laflamme [20]. Remarkably, even with vanishingly little entanglement, their approach can effectively handle some problems for which no effective classical algorithm is known [21]. The hunt for quantum correlations beyond entanglement, which should account for a quantum computer's efficiency, was spurred by this discovery. Since its introduction by Zurek in 2000, quantum discord has been acknowledged as a potential candidate for such general quantum correlations [22],[ 23]. On the one hand, non-entangled systems can also exhibit quantum discord. However, it has been demonstrated that Knill and Laflamme's approach displays a nonvanishing amount of discord [24]. He thought about how much work a classical and quantum Maxwell's demon could take from a quantum system. He demonstrated how the quantum demon is more potent since it can affect the entire quantum state, whereas the classical demon can only affect particular subsystems only [25],[26].

Henderson and Vedral developed a closely comparable quantity about the same time as Zurek established quantum discord [27]. By assuming a number of plausible characteristics, the authors sought to divide correlations into quantum and purely classical components. This strategy differs greatly from Zurek's, therefore it's unexpected that both produce the same outcome. The information deficit, as described in [28], is another related quantity. The writers Examine how much work can be extracted utilizing a mixed quantum state from a heat bath. When two parties share a mixed state, there is typically less extractable work than when one party owns the entire state. Given these findings, it should come as no surprise that jobs in quantum information theory that are not entanglement-based have received a great deal of attention in recent years [29]. Some of those tasks, such as remote state preparation [30] and information encoding[31] have been linked to quantum discord. There have also been experimental methods for identifying general quantum correlations[32].Within In this work, we provide an overview of broad quantum correlations that go beyond entanglement and provide a thorough analysis of their application to entanglement distribution [33],[34] , transmission of correlations [35], [36]. The mathematical foundation of quantum theory is first briefly reviewed, and then quantum entanglement is briefly introduced. Following the introduction of quantum discord and associated quantifiers of quantum correlations, we go over their significance in quantum information theory and provide a brief overview of other



research avenues.

## 2. Objectives

1. To explain the fundamentals of quantum theory and quantum entanglement.
2. To study quantum correlations beyond entanglement and their applications in quantum information processing.

## 3. Literature Review

Developed in the early 20th century to explain phenomena that classical mechanics was unable to describe, quantum theory is one of the most important and successful frameworks in contemporary physics. Max Planck (1900) introduced the idea of energy quantization in his explanation of blackbody radiation, laying the groundwork for quantum theory. Albert Einstein's explanation of the photoelectric phenomenon in 1905, which demonstrated how light behaves as discrete energy packets known as photons, further reinforced this theory. A new understanding of microscopic physical systems began with these works.

The separate formulations of Schrödinger's wave mechanics (1926) and Heisenberg's matrix mechanics (1925) led to the formal development of quantum mechanics. Born (1926) gave the probabilistic interpretation of the wavefunction, while Schrödinger introduced the wave equation that explains the evolution of a quantum system's wavefunction. When taken as a whole, these contributions demonstrated that quantum observations are statistical, which is a significant divergence from deterministic classical physics.

The superposition principle, which asserts that a quantum system can exist in several states concurrently until a measurement is made, is a significant outcome of quantum theory. Quantum entanglement, or non-classical correlations between particles, is directly caused by this principle. In their well-known EPR conundrum, which called into doubt the completeness of quantum physics, Einstein, Podolsky, and Rosen (1935) introduced the idea of



entanglement. They contended that correlations between entangled particles appear to go against local realism.

Erwin Schrödinger (1935) explicitly established the word entanglement in reaction to the EPR dilemma, characterizing it as a defining feature of quantum physics. Bell's inequalities were later developed by John Bell in 1964, which allowed for the experimental comparison of local hidden variable theories with the predictions of quantum mechanics. Bell's inequalities were definitively violated by Aspect et al. (1982) and later experiments, demonstrating the non-local character of quantum entanglement and bolstering the predictions of quantum theory.!

The practical significance of quantum entanglement in developing technologies has been highlighted in recent publications. According to studies by Bennett et al. (1993) and Ekert (1991), entanglement is a crucial resource in quantum computing, quantum cryptography, and quantum teleportation. Entanglement plays a crucial role in the development of quantum information science and is now being studied in condensed matter systems, quantum communication networks, and quantum metrology.

#### **4. Research Methodology**

Quantum theory adheres to a unique scientific methodology and is not only a collection of equations:

##### **(a) Description based on state**

A quantum state (wave function  $\psi$ ) describes every physical system.

All possible information about the system is contained in the state.

Properties don't have definitive values until they are measured, in contrast to traditional physics.

##### **(b) The principle of superposition**

A system can be in more than one state at once.

An electron, for instance, can exist in multiple locations simultaneously.

In terms of math:

$$|\psi\rangle=c_1|1\rangle+c_2|2\rangle$$

### (c) Observables and operators

Operators are used to represent physical quantities such as position, momentum, and energy. Applying an operator to a quantum state is equivalent to measurement.

### (d) Interpretation of probability

Probability amplitudes are used to forecast measurement results. Probability is obtained by squaring the wave function.

$$P=|\psi|^2$$

### e) Wave function collapse and measurement

The system is in superposition prior to measurement.

The system collapses to a specific state upon measurement.

Philosophical discussions result from this:

- >Copenhagen interpretation
- >Interpretation of many worlds
- >Theories of objective collapse

Quantum entanglement adheres to a unique scientific methodology :

### (a) Quantum theory's composite systems

Quantum theory uses a composite wave function to describe interactions between two or more particles:

$$|\psi\rangle_{\text{total}}\neq|\psi_1\rangle|\psi_2\rangle$$

### (a) Entanglement definition

When two particles are entangled:

It is impossible to divide their unified state into distinct states.

When one is measured, the other's condition is immediately ascertained.

For instance:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$$

### (c) Measurement techniques and entanglement

The entire entangled wave function collapses when one particle is measured.

Regardless of distance, this occurs.

□ This questioned traditional notions of:

Locality

Separate reality

It was described by Einstein as "*spooky action at a distance.*"

### (d) Relativity is not violated

Information cannot be sent more quickly than light through entanglement.

Usable signals do not surface instantly, but correlations do.

## 5. Quantum theory

### 5.1 Quantum states

A state vector  $|\Psi_i\rangle$  in a Hilbert space  $H$  fully characterizes any physical system in quantum mechanics. Another name for a system having a two-dimensional Hilbert space is a qubit (quantum bit). We consider a Hilbert space of arbitrary but finite dimension, unless otherwise noted. For two parties, Alice (A) and Bob (B), with Hilbert spaces  $H_A$  and  $H_B$  the total Hilbert space is a tensor product of the subsystem spaces:  $H_{AB} = H_A \otimes H_B$ .

A system is considered to be in a pure state if it can be described by a single state vector. Nevertheless, the physical state of the system under consideration is not entirely understood in a true experimental setup. If the system has probability  $p_i$  and is in the pure state  $|\psi_{ii}\rangle$ . The density operator can be used to describe the system's physical condition.

$$\rho = \sum_i p_i |\psi_{ii}\rangle \langle \psi_{ii}|. \quad (5.1)$$

Such a system is said to be in a mixed state. When we refer to quantum states in the following, we typically mean mixed states. Any density operator must have the following two

properties in order to have a valid physical interpretation:

- $\rho$  has trace equal to one:

$$\text{Tr}[\rho] = 1,$$

(5.2)

- $\rho$  is a positive operator:

$$\langle \psi | \rho | \psi \rangle \geq 0$$

(5.3)

It should be noted that the second property also suggests that  $\rho$  is Hermitian:  $\rho = \rho^\dagger$ . The definition of quantum measurements and operations, which is provided in the following.

## 5.2 Quantum measurements and operations

Quantum measurement is one of the most important concepts in quantum theory. Most physicists are familiar with the projective measurement: for a spin-1/2 particle in the state

$$|\psi\rangle = a|\uparrow\rangle + b|\downarrow\rangle,$$

(5.4)

The probability to measure “spin up” or “spin down” is given by  $p(\uparrow) = |a|^2$  or  $p(\downarrow) = |b|^2 = 1 - p(\uparrow)$ . Moreover, the measurement postulate of quantum mechanics tells us that the quantum state after the measurement is either  $|\uparrow\rangle$  or  $|\downarrow\rangle$ , depending on the outcome of the measurement.

In quantum information theory, a more general definition is considered. A general quantum measurement is described by a collection  $\{E_i\}$  of measurement operators that satisfy the completeness equation:

$$\sum_i E_i^\dagger E_i = 1,$$

(5.5)

Where 1 is the identity operator. Given a density operator  $\rho$  and the set of measurement operators  $\{E_i\}$ , the probability that the result  $i$  occurs is given by

$$\rho_i = \text{Tr}[E^\dagger_i \rho].$$

(5.6)

After the measurement with outcome  $i$ , the state of the system is described by the density operator

$$\rho_i = \frac{1}{p_i} (E_i \rho E_i^\dagger).$$

(5.7)

The set of operators

$$M_i = E_i^\dagger E_i$$

(5.8)

is also called positive operator-valued measure (POVM). Due to the completeness equation (5.5), the POVM elements  $M_i$  sum up to the identity operator:  $\sum_i M_i = 1$ . Moreover, due to Eq. (5.6) the probabilities  $p_i$  can also be obtained from the POVM elements  $M_i$ :  $p_i = \text{Tr}[M_i \rho]$ . The positivity of the density operator  $\rho$  in Eq. (5.3) implies that all probabilities are nonnegative:  $p_i \geq 0$ . The completeness equation (5.5) together with Eq. (5.2) implies that the probabilities sum up to one:  $\sum_i p_i = 1$ .

For a projective measurement, the operators  $E_i$  are orthogonal projectors:  $E_i E_j = \delta_{ij} E_i$ . Von Neumann measurement is a special type of a projective measurement, where the measurement operators  $E_i$  are orthogonal projectors with rank one. Such a measurement was considered below Eq. (5.4), there the measurement operators are  $E_\uparrow = |\uparrow\rangle\langle\uparrow|$  and  $E_\downarrow = |\downarrow\rangle\langle\downarrow|$ . In general, the measurement operators do not have to be projectors, they only need to satisfy the completeness equation (5.5).

For composite systems consisting of two subsystems, Alice and Bob, it is possible to perform local measurements on one of the subsystems. If a local measurement is done on Alice's subsystem, the subsystem of Bob remains unchanged. In this case, the measurement operators have the form  $E_i = E_A^i \otimes 1_B$ , with the identity operator  $1_B$  on Bob's Hilbert space. Similarly, measurement operators corresponding to local measurement on Bob's subsystem have the form  $E_i = 1_A \otimes E_B^i$ .

Finally, we also mention the concept of quantum operations, which is closely related to quantum measurements. Any set of measurement operators  $\{E_i\}$  can also be called a quantum operation. The corresponding operators  $E_i$  are then called Kraus operators. The action of a quantum operation  $\{E_i\}$  on a density operator  $\rho$  is given by

$$\Lambda (\rho) = \sum_i E_i \rho E_i^\dagger \quad (5.9)$$

Local quantum operations for composite systems can be specified in the same manner as local measurements. Because they describe the most general modification of a quantum state that can be achieved in experiments, quantum operations are significant. Additionally, quantum processes are crucial to the study of noisy systems: Typically, noise is represented as a quantum operation.

### 5.3 Reduced density operator

Sometimes one is only interested in one of the subsystems of a composite quantum system. This situation is captured by the concept of the reduced density operator. If the total system is described by the density operator  $\rho_{AB}$ , then the system of A is described by the reduced density operator.

$$\rho_A = \text{Tr}_B[\rho_{AB}],$$

(5.10)

Where  $\text{Tr}_B$  is called partial trace over the subsystem B. The partial trace is defined by

$$\text{Tr}_B[|a_1\rangle\langle a_2| \otimes |b_1\rangle\langle b_2|] = |a_1\rangle\langle a_2| \text{Tr}[|b_1\rangle\langle b_2|], \quad (5.11)$$

where  $|a_1\rangle$  and  $|a_2\rangle$  are any two vectors in  $H_A$ , and  $|b_1\rangle$  and  $|b_2\rangle$  are any two vectors in  $H_B$ . The trace on the right hand side is the usual trace for the subsystem B:  $\text{Tr}[|b_1\rangle\langle b_2|] = \langle b_2|b_1\rangle$ . In addition to Eq. (5.11), we also require that the partial trace is linear, i.e.,  $\text{Tr}_B[M_{AB} + N_{AB}] = \text{Tr}_B[M_{AB}] + \text{Tr}_B[N_{AB}]$  for any two operators  $M_{AB}$  and

N AB. In this way, the partial trace is defined for all density operators. The physical meaning of the partial trace lies in the fact that it is the unique operation for obtaining correct measurement statistics for the subsystem A.

#### 5.4 Entropy and mutual information

The von Neumann entropy of a quantum state with density operator  $\rho$  is defined as

$$S(\rho) = -\text{Tr}[\rho \log_2 \rho], \quad (5.12)$$

where the logarithm of the density operator  $\rho$  is defined via its Eigen values  $\lambda_i$  and eigenstates  $|i\rangle$  in the following way:  $\log_2 \rho = \sum_i \log_2(\lambda_i) |i\rangle\langle i|$ . With this definition, the entropy can be written as

$$S(\rho) = -\sum_i \lambda_i \log_2 \lambda_i, \quad (5.13)$$

where it is defined that  $0 \log_2 0 = 0$ .

The von Neumann entropy is the quantum version of the classical Shannon entropy. For a discrete random variable  $X$  which can take a value  $x$  with probability  $p_x$ , the Shannon entropy is defined as

$$H(X) = -\sum_x p_x \log_2 p_x. \quad (5.14)$$

The von Neumann entropy quantifies the uncertainty of a quantum state, much like the Shannon entropy does for a classical random variable. Since their von Neumann entropy is zero, pure states signify complete understanding of a quantum system. However, in contrast, the completely mixed density operator  $1/d$  with the von Neumann entropy  $\log_2 d$  represents maximal uncertainty for a  $d$ -dimensional Hilbert space. The mutual information between two parties can be defined using the von Neumann entropy. The mutual information is defined as follows if the density operator  $\rho_{AB}$  provides the complete state with reduced density operators  $\rho_A$  and  $\rho_B$ .

$$I(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB}). \quad (5.15)$$

The mutual information is zero if the state is completely uncorrelated, i.e., if the density operator has the form  $\rho_{AB} = \rho_A \otimes \rho_B$ . Otherwise, the mutual information is greater than zero: it measures the amount of correlations between A and B. Closely related to the von Neumann entropy is the quantum relative entropy. For two density operators  $\rho$  and  $\sigma$  it is defined as

$$S(\rho||\sigma) = \text{Tr}[\rho \log_2 \rho] - \text{Tr}[\rho \log_2 \sigma] \quad (5.16)$$

if the support of  $\rho$  is contained in the support of  $\sigma$ , and  $S(\rho||\sigma) = +\infty$  otherwise. The quantum relative entropy is nonnegative, and zero if and only if  $\rho = \sigma$ . The mutual information defined in Eq. (5.15) can be written as the relative entropy between the density operator  $\rho_{AB}$  and the tensor product of the reduced density operators  $\rho_A \otimes \rho_B$  [37]:

$$I(\rho_{AB}) = S(\rho_{AB}||\rho_A \otimes \rho_B). \quad (5.17)$$

## 5.5 Distance between density operators

Given two quantum states, how “close” are they to each other? This question, can be answered by defining an appropriate distance onto the set of density operators. One important and frequently used distance is the trace distance

$$D_t(\rho, \sigma) = \frac{1}{2} \text{Tr}|\rho - \sigma|, \quad (5.18)$$

Where  $\rho$  and  $\sigma$  are any two density operators,  $\text{Tr}|M| = \text{Tr} \sqrt{M^\dagger M}$  is the trace norm of an operator  $M$ , and the square root of a Hermitian operator  $M^\dagger M$  with nonnegative eigenvalues  $\lambda_i$  and eigenstates  $|i\rangle$  is defined as  $\sqrt{M^\dagger M} = \sum_i \sqrt{\lambda_i} |i\rangle \langle i|$ . The trace distance satisfies all properties of a general distance  $D$ :

- $D(\rho, \sigma) \geq 0$ , and  $D(\rho, \sigma) = 0$  holds if and only if  $\rho = \sigma$ ,
- $D$  is symmetric:  $D(\rho, \sigma) = D(\sigma, \rho)$ ,
- $D$  satisfies the triangle inequality:  $D(\rho, \tau) \leq D(\rho, \sigma) + D(\sigma, \tau)$  for any three density operators  $\rho, \sigma$ , and  $\tau$ .

In quantum information theory, the trace distance has an important interpretation:  $\frac{1}{2} \|\rho - \sigma\|_1$  is the optimal probability of success for distinguishing two quantum states with density operators  $\rho$  and  $\sigma$  [38]. Another frequently used quantity is the fidelity. For two density operators  $\rho$  and  $\sigma$  it is defined as

$$F(\rho, \sigma) = \text{Tr} \sqrt{\sqrt{\rho} \sigma \sqrt{\rho}} \quad (5.19)$$

The fidelity itself is not a distance, since it is one if and only if  $\rho = \sigma$ , and smaller than one otherwise. However, the fidelity can be used to define the Bures distance:  $D_B(\rho, \sigma) = \sqrt{2(1 - F(\rho, \sigma))}$ , which satisfies all properties of a mathematical distance.

Both, the trace distance and the Bures distance have also another important property, namely they are non increasing under quantum operations:

$$D(\Lambda(\rho), \Lambda(\sigma)) \leq D(\rho, \sigma), \quad (5.20)$$

where  $\Lambda$  is any quantum operation and  $\rho$  and  $\sigma$  are any two density operators. In quantum information theory, this feature is often employed, particularly when examining entanglement and other quantum correlations.

It should be noted that there are distances that contradict the inequality (5.20) since it does not follow from the general features of a mathematical distance. The Hilbert-Schmidt distance is one such distance.

$$D_{HS}(\rho, \sigma) = \sqrt{\text{Tr}(\rho - \sigma)^2} \quad (5.21)$$

where  $\|M\|_2 = \sqrt{\text{Tr}[M^\dagger M]}$  is the Hilbert-Schmidt norm of an operator  $M$ . For the Hilbert Schmidt distance violation of Eq. (5.20) was shown in [39], [40].

Lastly, because it is not symmetric and does not meet the triangle inequality, the relative entropy created in Eq. (5.16) is not a distance in the mathematical sense. Nevertheless, under quantum processes, the relative entropy is non increasing, meaning it meets inequality (5.20) [41].

## 6. Quantum entanglements

## 6.1 Definition

For two parties, Alice (A) and Bob (B), the state of the total quantum system can have product form:

$$|\Psi_i = |a_i \otimes |b_i,$$

where the associated local Hilbert spaces  $H_A$  and  $H_B$  include the states  $|a_i$  and  $|b_i$ . States of the type described in Eq. (6.1) are also referred to as separable since they are not entangled. But since quantum mechanics also permits superpositions that are not separable, not all states are unavoidable product:

$$|\Phi_i = \frac{1}{\sqrt{2}} (|a_{1i} \otimes |b_{1i} + |a_{2i} \otimes |b_{2i}),$$

where  $N$  assures normalization such that  $\langle \Phi | \Phi \rangle = 1$ . If  $|\Phi_i$  cannot be written as a product, i.e.,  $|\Phi_i, |a_i \otimes |b_i$ , the state is called entangled.

Example: The singlet state  $|\Phi_i = \frac{1}{\sqrt{2}} (|01i - |10i)$  is entangled, it cannot be written as a product.

A mixed state is separable if it can be written as a convex combination of pure product states:

$$\rho_{sep} = \sum_i p_i |a_{ii} \rangle \langle a_{ii}| \otimes |b_{ii} \rangle \langle b_{ii}|.$$

The local Hilbert spaces  $H_A$  and  $H_B$  contain the pure states  $|a_{ii}$  and  $|b_{ii}$ , and  $p_i \geq 0$  are probabilities that add up to one:  $\sum_i p_i = 1$ . In the event that the state cannot be expressed in this way, it is known as intertwined.

This concept of entanglement is based on the following premise: let's say Alice and Bob can locally create any quantum state. They also have access to a traditional means of communication, like a phone. Then, using the process described in Eq. (6.3), Alice and Bob can create any separable state: Alice uses the probability  $p_i$  to prepare the state  $|a_{ii}$ , and she informs Bob of the state she prepared. Bob prepares the appropriate state based on this information. , Bob prepares the corresponding state  $|b_{ii}$ . On the other hand, it is not possible to create entangled states such as the singlet state in this way.



## 6.2 Local operations and classical communication

The method for generating separable states described above is a member of the local operations and classical communication (LOCC) class, which was first published in [42]. If Alice and Bob are restricted to classical communication alone, this class of operations represents the most general process they may use in quantum theory. The whole mathematical explanation of these procedures is difficult and yet the focus of a lot of research [43]. Nonetheless, the overall concept is straightforward and will be clarified as follows. A quantum operation  $\Psi$ LOCC for two parties, Alice and Bob, is in the LOCC class if it can be broken down into the subsequent steps:

1. A local measurement is carried out on Alice's subsystem by one of the parties.
2. Bob, the opposite party, is informed of the measurement's result in a traditional manner.
3. Bob measures his subsystem locally based on the data he has received.
4. Alice receives the results of Bob's measurement in a traditional manner.
5. Alice measures her subsystem locally based on the information she has received, and the procedure repeats at step 2. In quantum information theory, the class of LOCC is crucial, particularly when examining entanglement. As previously stated, LOCC can be used to produce any separable state. However, entangled states cannot be produced via LOCC.

## 6.3 Entanglement as a resource

Prior to the 1990s, quantum entanglement was primarily thought of as a physics curiosity—an unusual characteristic with no real-world applications. When Ekert introduced the first entanglement-based task in quantum information theory in 1991, things began to shift. In his work, Ekert demonstrated that two parties, Alice and Bob, can communicate in a totally safe manner if they share a significant number of entangled singlet states. This process is known as quantum key distribution, or quantum cryptography. This impressive outcome should be contrasted with modern classical cryptography.

Inspired by Ekert's finding, a number of entanglement-related activities have been introduced in subsequent years. Bennett and Wiesner demonstrated in 1992 that two entangled parties could exchange two classical bits using just one qubit, or one quantum system on a two-dimensional Hilbert space. Because it implies that two classical bits can be coded into one quantum bit, this activity is also known as quantum dense coding. An additional use for

entanglement has been suggested. The writers examined the assignment. of two parties communicating an unknown quantum state. Since an unknown quantum state cannot be duplicated, it cannot be conveyed by classical methods [44].

However, Bennett et al. demonstrated that any unknown quantum bit can be perfectly transmitted if the two parties share an entangled singlet. Another name for this task is quantum teleportation.

#### **6.4 Entanglement measures**

The aforementioned tasks—quantum cryptography, dense coding, and teleportation—showcase the function of entanglement in a particularly specific scenario. Specifically, to accomplish these duties, two parties, Alice and Bob,

must share entangled singlets. Nevertheless, a pure quantum state is not always a singlet, and in practical situations, the quantum state is typically mixed. For Because of this, it is reasonable to wonder if some of these activities can

also be performed using a general mixed quantum state. The degree of entanglement in a quantum state is typically used to gauge its "usefulness" for one of the aforementioned tasks.

The distillable entanglement is one of the most

widely used quantifiers [45]: It is defined as the maximum number of singlets that, if the number of copies increases to infinity, may be acquired per copy of a given mixed state using local operations and classical communication. The primary drawback of distillable entanglement is its difficulty in assessment. As a result, only a few unique situations are known to have exact expressions. Other quantifiers, referred to as entanglement measures, have been put out in the literature as a result. The following two characteristics are satisfied by any entanglement measure:

1. Under local operations and classical communication, E does not rise;
2. On separable states, E disappears.

For a pure state  $|\psi\rangle_{AB}$  distributed between two parties, Alice and Bob, entanglement is usually quantified by the von Neumann entropy of the reduced density operator  $\rho_A = \text{Tr}_B[|\psi\rangle\langle\psi|_{AB}]$ :

$$E(|\psi\rangle_{AB}) = S(\rho_A) = -\sum_i \lambda_i \log_2 \lambda_i, \quad (6.4)$$

The significance of this quantity in quantum information, where  $\lambda_i$  are the eigenvalues of  $\rho_A$  idea stems from the fact that for all pure states, it is equivalent to the distillable entanglement [46].

The literature considers two major groups of entanglement measurements for a mixed state  $\rho_{AB}$ . Convex roof measures and distance-based measures are two of them. The following convex roof architecture can be used to extend any measure of entanglement  $E$  defined on all pure states to mixed states [47]:

$$E(\rho) = \inf \{ \sum_i p_i E(|\psi_i\rangle) \} \quad (6.5)$$

Where the infimum is taken over all decompositions  $\{p_i, |\psi_i\rangle\}$  of the given density operator  $\rho$  with nonnegative probabilities  $p_i$ , i.e.,

$$\rho = \sum_i p_i |\psi_i\rangle \langle \psi_i|.$$

One of the most widely used and well-liked convex roof metrics for bipartite systems is the entanglement of formation defined. It is defined as the von Neumann entropy of the reduced density operator in Eq. (6.4) for pure states. The process of extending to mixed states involves the construction of the convex roof in Eq. (6.5). Wootters provided a closed expression for the entanglement of formation for all mixed states of two qubits [48], despite the fact that the infimum in Eq. (6.5) is generally difficult to estimate. The entanglement of formation  $E_f$  for any such state is determined by

$$E_f(\rho) = h\left(\frac{1}{2} + \frac{1}{2} \sqrt{1 - C(\rho)^2}\right) \quad (6.6)$$

with the binary entropy  $h(x) = -x \log_2 x - (1-x) \log_2 (1-x)$ , and the concurrence  $C(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$ , where  $\lambda_i$  are the square roots of the eigenvalues of  $\rho \tilde{\rho}$  in decreasing order, and  $\tilde{\rho}$  is defined as  $\tilde{\rho} = (\sigma_y \otimes \sigma_y) \rho (\sigma_y \otimes \sigma_y)$  with the Pauli matrix  $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ . It does not increase under local operations and classical communication and

vanishes on separable states. While the second property is easy to verify, the first property was proven.

The second main class of entanglement measures are measures based on distance proposed in [49]. All those measures can be written as

$$E(\rho) = \inf_{\sigma \in S} D(\rho, \sigma), \quad (6.7)$$

where  $D$  is a distance and the infimum is taken over the set of separable states  $S$ . If the distance  $D$  does not increase under quantum operations, i.e.,

$$D(\Lambda(\rho), \Lambda(\sigma)) \leq D(\rho, \sigma) \quad (6.8)$$

for any quantum operation  $\Lambda$  and any two states  $\rho$  and  $\sigma$ , then the corresponding measure of entanglement does not increase under local operations and classical communication [49]. This property is satisfied by the relative entropy  $S(\rho||\sigma) = \text{Tr}[\rho \log_2 \rho] - \text{Tr}[\rho \log_2 \sigma]$ , although the relative entropy is not a distance in the mathematical sense. The corresponding measure of entanglement is called relative entropy of entanglement:

$$ER(\rho) = \min_{\sigma \in S} S(\rho||\sigma). \quad (6.9)$$

One of the most well-liked and extensively researched measurements of entanglement is the relative entropy. One explanation is because quantum information theory heavily relies on the relative entropy itself. Additionally, the relative entropy of entanglement is a potent distillable entanglement upper bound. As we previously said, if the distance fulfills Eq. (6.8), then all distance-based entanglement measures do not rise under local operations and classical transmission. Any realistic measure of entanglement should meet this requirement. Furthermore, in separable states, any entanglement measure ought to disappear as well. This is also clearly true for any distance

$D(\rho, \sigma)$ , which is greater than zero otherwise and zero if and only if  $\rho = \sigma$ .

Lastly, we discuss the relationship between three of the metrics discussed in this section: the entanglement of formation ( $E_f$ ), the relative entropy of entanglement ( $ER$ ), and the distillable entanglement ( $E_d$ ). These measurements satisfy the inequality, as demonstrated in [50].

$$E_d \leq E_R \leq E_f$$

(6.10)

for all mixed states, i.e., the relative entropy of entanglement is always between  $E_d$  and  $E_f$ .

## 7. DISCUSSION

When compared to entangled states, Dakic et al. find that a shared separable state can perform better for remote state preparation. Specifically, if Alice and Bob adhere to the protocol exactly—that is, if Alice does von Neumann measurements and Bob conditionally. There are situations when shared states without any entanglement can perform better than shared entangled states when a  $\pi$  rotation is applied around a specific axis. Dakicé et al. established a reward function  $P$  as a measure of the process's performance and shown that, in many cases, the average optimal payment is constrained below by the geometric measure of discord.

Dakic et al. established a reward function  $P$  as a measure of the process's performance and shown that, in many cases, the average optimal payment is bounded below by the geometric measure of discord. Discord ensures that remote state preparation can always be accomplished with a nonzero average payout under these circumstances. There have also been reports of experiments that support similar findings.

We conclude by discussing the current critique of this methodology. On the one hand, it was demonstrated in that a state can result in a nonzero average reward even if local noise caused its discord. Such states are unlikely to be helpful in quantum information theory, according to. However, the protocol's limitation to Alice's von Neumann measurements and Bob's conditional rotations was criticized in. This matter was further examined in, where it was demonstrated that by easing these limitations, the benefit of if the standard fidelity  $(1 + r^2 s)/2$  is utilized as the protocol's figure of merit, separable states vanish. Nevertheless that, under certain circumstances, separable states might still be advantageous for remote state preparation with the standard fidelity  $(1 + r^2 s)/2$ .

## 8. CONCLUSION

Quantum entanglement is one of the most significant and scientifically confirmed effects of quantum theory, which offers the basic foundation for comprehending microscopic events. The literature demonstrates a distinct progression from fundamental discussions to real-world applications. Our comprehension of nature at the microscopic level has been profoundly altered by quantum theory. It demonstrates that, in contrast to conventional physics, particles like electrons and photons do not have precise locations or energy until they are detected. Rather, probability waves characterize their behavior, emphasizing the inherent uncertainty and wave-particle duality seen in the quantum universe.

One of the most amazing outcomes of quantum theory is quantum entanglement. It shows that no matter how far apart two or more particles are from one another, they can become correlated to the point where their states are instantly connected. This non-classical relationship has been experimentally confirmed, demonstrating the completeness of quantum mechanics, and it cannot be described by classical physics.

## 9. Acknowledgement

For giving me the chance and academic setting to do this dissertation on quantum theory and quantum entanglement, I would want to sincerely thank Carrer Point University. I am incredibly grateful to my esteemed faculty members for their invaluable advice, unwavering support, and perceptive recommendations during this project.

I also want to express my gratitude to the university administration and library personnel for providing the facilities and resources needed for this endeavor. Lastly, I want to express my gratitude to my friends and classmates for their support and collaboration, which helped me finish this project successfully.

## REFERENCES

1. Streltsov, A. (2014). Quantum correlations beyond entanglement. In *Quantum Correlations Beyond Entanglement: and Their Role in Quantum Information Theory* (pp. 17-22). Cham: Springer International Publishing.
2. Schrödinger, E. (1935). Die gegenwärtige Situation in der Quantenmechanik. *Naturwissenschaften*, 23(50), 844-849.

3. Nielsen, M. A., & Chuang, I. L. (2010). *Quantum computation and quantum information*. Cambridge university press.
4. Einstein, A., Podolsky, B., & Rosen, N. (1935). Can quantum-mechanical description of physical reality be considered complete *Physical review*, 47(10), 777.
5. Bell, J. S. (1964). On the einstein podolsky rosen paradox. *Physics Physique Fizika*, 1(3), 195.
6. Clauser, J. F., Horne, M. A., Shimony, A., & Holt, R. A. (1969). Proposed experiment to test local hidden-variable theories. *Physical review letters*, 23(15), 880.
7. Freedman, S. J., & Clauser, J. F. (1972). Experimental test of local hidden-variable theories. *Physical review letters*, 28(14), 938.
8. Aspect, A., Grangier, P., & Roger, G. (1981). Experimental tests of realistic local theories via Bell's theorem. *Physical review letters*, 47(7), 460.
9. Aspect, A., Dalibard, J., & Roger, G. (1982). Experimental test of Bell's inequalities using time-varying analyzers. *Physical review letters*, 49(25), 1804.
10. Aspect, A., Grangier, P., & Roger, G. (1982). Experimental realization of Einstein-Podolsky-Rosen-Bohm Gedankenexperiment: a new violation of Bell's inequalities. *Physical review letters*, 49(2), 91.
11. Horodecki, R. H. (2009). M., & Horodecki K. *Quantum entanglement. Rev. Mod. Phys*, 81, 865.
12. Werner, R. F. (1989). Quantum states with Einstein-Podolsky-Rosen correlations admitting a hidden-variable model. *Physical Review A*, 40(8), 4277.
13. Ekert, A. K. (1991). Quantum cryptography based on Bell's theorem. *Physical review letters*, 67(6), 661.
14. Bennett, C. H., & Wiesner, S. J. (1992). Communication via one-and two-particle operators on Einstein-Podolsky-Rosen states. *Physical review letters*, 69(20), 2881.
15. Bennett, C. H., Brassard, G., Crépeau, C., Jozsa, R., Peres, A., & Wootters, W. K. (1993). Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. *Physical review letters*, 70(13), 1895.
16. Horodecki, M., Horodecki, P., & Horodecki, R. (1998). Mixed-state entanglement and distillation: Is there a "bound" entanglement in nature?. *Physical Review Letters*, 80(24), 5239.



17. Jozsa, R. (1997). Entanglement and quantum computation. *arXiv preprint quant-ph/9707034*.
18. Jozsa, R., & Linden, N. (2003). On the role of entanglement in quantum-computational speed-up. *Proceedings of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences*, 459(2036), 2011-2032.
19. Shor, P. W. (1994, November). Algorithms for quantum computation: discrete logarithms and factoring. In *Proceedings 35th annual symposium on foundations of computer science* (pp. 124-134). Ieee.
20. Knill, E., & Laflamme, R. (1998). Power of one bit of quantum information. *Physical Review Letters*, 81(25), 5672.
21. Datta, A., Flammia, S. T., & Caves, C. M. (2005). Entanglement and the power of one qubit. *Physical Review A—Atomic, Molecular, and Optical Physics*, 72(4), 042316.
22. Zurek, W. H. (2000). Einselection and decoherence from an information theory perspective. *Annalen der Physik*, 512(11-12), 855-864.
23. Ollivier, H., & Zurek, W. H. (2001). Quantum discord: a measure of the quantumness of correlations. *Physical review letters*, 88(1), 017901.
24. Datta, A., Shaji, A., & Caves, C. M. (2008). Quantum discord and the power of one qubit. *Physical review letters*, 100(5), 050502.
25. Eastin, B. (2010). Simulating concordant computations. *arXiv preprint arXiv:1006.4402*.
26. Zurek, W. H. (2003). Quantum discord and Maxwell's demons. *Physical Review A*, 67(1), 012320.
27. Henderson, L., & Vedral, V. (2001). Classical, quantum and total correlations. *Journal of physics A: mathematical and general*, 34(35), 6899.
28. Oppenheim, J., Horodecki, M., Horodecki, P., & Horodecki, R. (2002). Thermodynamical approach to quantifying quantum correlations. *Physical review letters*, 89(18), 180402.
29. Modi, K., Brodutch, A., Cable, H., Paterek, T., & Vedral, V. (2012). The classical-quantum boundary for correlations: discord and related measures. *Reviews of Modern Physics*, 84(4), 1655-1707.
30. Dakić, B., Lipp, Y. O., Ma, X., Ringbauer, M., Kropatschek, S., Barz, S., ... & Walther, P. (2012). Quantum discord as resource for remote state preparation. *Nature*



*Physics*, 8(9), 666-670.

31. Gu, M., Chrzanowski, H. M., Assad, S. M., Symul, T., Modi, K., Ralph, T. C., ... & Lam, P. K. (2012). Observing the operational significance of discord consumption. *Nature Physics*, 8(9), 671-675.
32. Gessner, M., Ramm, M., Pruttivarasin, T., Buchleitner, A., Breuer, H. P., & Häffner, H. (2014). Local detection of quantum correlations with a single trapped ion. *Nature Physics*, 10(2), 105-109.
33. Streltsov, A., Kampermann, H., & Bruß, D. (2012). Quantum cost for sending entanglement. *Physical review letters*, 108(25), 250501.
34. Chuan, T. K., Maillard, J., Modi, K., Paterek, T., Paternostro, M., & Piani, M. (2012). Quantum discord bounds the amount of distributed entanglement. *Physical Review Letters*, 109(7), 070501.
35. Streltsov, A., & Zurek, W. H. (2013). Quantum discord cannot be shared. *Physical review letters*, 111(4), 040401.
36. Brandao, F. G., Piani, M., & Horodecki, P. (2015). Generic emergence of classical features in quantum Darwinism. *Nature communications*, 6(1), 7908.
37. Vedral, V. (2002). The role of relative entropy in quantum information theory. *Reviews of Modern Physics*, 74(1), 197.
38. Fuchs, C. A., & Van De Graaf, J. (2002). Cryptographic distinguishability measures for quantum-mechanical states. *IEEE Transactions on Information Theory*, 45(4), 1216-1227.
39. Ozawa, M. (2000). Entanglement measures and the Hilbert–Schmidt distance. *Physics Letters A*, 268(3), 158-160.
40. Piani, M. (2012). Problem with geometric discord. *Physical Review A—Atomic, Molecular, and Optical Physics*, 86(3), 034101.
41. Lindblad, G. (1975). Completely positive maps and entropy inequalities. *Communications in Mathematical Physics*, 40(2), 147-151.
42. Bennett, C. H., DiVincenzo, D. P., Smolin, J. A., & Wootters, W. K. (1996). Mixed-state entanglement and quantum error correction. *Physical Review A*, 54(5), 3824.
43. Chitambar, E., Leung, D., Mančinska, L., Ozols, M., & Winter, A. (2014). Everything you always wanted to know about LOCC (but were afraid to ask). *Communications in Mathematical Physics*, 328(1), 303-326.



44. Wootters, W. K., & Zurek, W. H. (1982). A single quantum cannot be cloned. *Nature*, 299(5886), 802-803.
45. Bennett, C. H., Brassard, G., Popescu, S., Schumacher, B., Smolin, J. A., & Wootters, W. K. (1996). Purification of noisy entanglement and faithful teleportation via noisy channels. *Physical review letters*, 76(5), 722.
46. Bennett, C. H., Bernstein, H. J., Popescu, S., & Schumacher, B. (1996). Concentrating partial entanglement by local operations. *Physical Review A*, 53(4), 2046.
47. Uhlmann, A. (1998). Entropy and optimal decompositions of states relative to a maximal commutative subalgebra. *Open Systems & Information Dynamics*, 5(3), 209-228.
48. Wootters, W. K. (1998). Entanglement of formation of an arbitrary state of two qubits. *Physical Review Letters*, 80(10), 2245.
49. Vedral, V., Plenio, M. B., Rippin, M. A., & Knight, P. L. (1997). Quantifying entanglement. *Physical Review Letters*, 78(12), 2275.
50. Horodecki, M., Horodecki, P., & Horodecki, R. (2000). Limits for entanglement measures. *Physical Review Letters*, 84(9), 2014.