



Nonlinear Dynamics of Intense Laser Pulses Reflecting off Sliding Plasma Layers

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Abstract: The nonlinear interaction of an intense relativistic laser pulse incident obliquely on a thin overdense plasma foil in the radiation pressure dominant (RPD) domain is studied analytically and numerically. Because of the strong electric field produced by the remaining ions, the density of the plasma foil is so large that the electrons contained inside it simply cannot fly from the plasma surface. Nevertheless, the ions and electrons are free to move around on the surface of the foil, and so form a sliding plasma mirror. The amplitude of the reflected laser pulse versus the incident laser frequency and the density of the plasma foil is demonstrated numerically in this study. We also report our numerical findings for the accelerated ions' energy and reflection coefficient as a function of laser frequency.

Keywords: Radiation pressure dominant regime, Nonlinear interaction, Relativistic laser pulse, Plasma mirror, Ion acceleration.

I. INTRODUCTION

The interaction of an ultra-intense laser of intensity $I \sim 10^{22} \text{ W/cm}^2$, with an ultrathin, overdense metallic foil triggers a transition into the relativistic regime, effectively converting the solid target into a high-density plasma mirror [1]–[5]. The foil serves as a high-fidelity mirror for the incoming rays in this regime [2]. Due to the elevated plasma density, the electrons remain trapped within the restorative electrostatic potential of the ion layer. The incident laser field, unable to overcome this confinement, drives the electrons into complex nonlinear oscillations transverse to the foil surface, establishing a sliding mirror configuration [6]. At laser intensities exceeding 10^{22} W/cm^2 , relativistic nonlinearities become the dominant factor in the light-matter interaction. Significant frequency upshifting, high-order harmonic generation (HHG), and the creation of isolated attosecond pulses upon reflection



are all made possible by these dynamics. A fundamental governing parameter for this

interaction is the normalized plasma density defined as: $\epsilon_p = \frac{\int \omega_p^2(x) dx}{2\omega_0 c}$. Here

$\omega_p = (4\pi n e^2 / m)^{1/2}$ is the plasma frequency, ω_0 is the angular frequency of the laser, n denotes electron density, m referred as the electron mass, e representing electron charge and c is the vacuum speed of light. For a plasma of uniform density, this dimensionless parameter is expressed as: $\epsilon_p = (\pi n l) / (n_{cr} \lambda_0)$, where l and λ_0 stands for the plasma foil thickness, and laser wavelength, respectively, and $n_{cr} = m\omega_0^2 / (4\pi e^2)$ represents the critical electron density. The transition between the transparent and opaque regimes is strictly dictated by the magnitude of ϵ_p . Specifically, the plasma foil is transparent to the incident pulse when $\epsilon_p \ll 1$ and transitions to a reflective, opaque state for $\epsilon_p \gg 1$.

The coordinate transformations from the laboratory frame to the relativistic boosted frame are presented in Section 2 of this study. This frame makes it easier to treat the moving mirror analytically by simplifying the laser's oblique incidence to a normal incidence geometry. Section 3 develops the theoretical model by solving the wave equations subject to nonlinear boundary conditions. This section explores the underlying physics of HHG and the synthesis of attosecond pulses. Section 4 presents a detailed analysis of the numerical results, discussing the influence of various laser-plasma parameters on the reflection spectra. A summary of the findings and the final conclusions are presented in the last section.

II. THEORETICAL FRAMEWORK AND BOOSTED FRAME TRANSFORMATION

As illustrated in Fig. 1 of reference [6], consider a plane electromagnetic wave in vacuum that is obliquely incident on the thin plasma foil at an angle with respect to the surface normal (x-axis). To simplify the interaction dynamics, we utilize Lorentz transformation equations to map the laboratory frame onto a relativistic boosted frame moving parallel to the foil surface (y-axis) [7]. We define a boosted reference frame moving with velocity along the y-direction. Under this transformation, the frequency and the wave vector components are transformed according to:

$$\omega'_0 = \gamma(\omega_0 - k_y V), \quad k'_x = k_x, \quad k'_y = \gamma(k_y - \omega_0 V / c^2),$$

(1) where $\gamma = (1 - V^2 / c^2)^{-1/2}$ denotes the Lorentz factor and primed quantities denote



values in the boosted frame. By setting the boost velocity to $V = k_y c^2 / \omega_0 = c \sin \theta$, the y -component of the wave vector k_y' vanishes, reducing the problem to a normal incidence geometry where the wave propagates solely along the x' -axis. For an s -polarized wave, the electric field in the laboratory frame is oriented normal to the plane of incidence (along the z -axis), while the magnetic field resides in the xy -plane. If the electric field expressed in the laboratory frame is [6] $\vec{E}_0 = E_s(t_r) \hat{e}_z$, then applying the Lorentz transformation for the field tensors, the incident electric and magnetic fields in the boosted frame are expressed as:

$$\vec{E}'_0 = E_s(t_r \cos \theta) \cos \theta \hat{e}_z \quad \text{and} \quad \vec{B}'_0 = E_s(t_r \cos \theta) \cos \theta \hat{e}_y,$$

(2) where E_s is the amplitude of the s -polarized pulse, $t_r = t - \vec{n} \cdot \vec{r} / c$, $\vec{n} = \cos \theta \hat{e}_x + \sin \theta \hat{e}_y$ is the unit vector along the propagation direction, $\vec{r} = x \hat{e}_x + y \hat{e}_y + z \hat{e}_z$ and $t'_r = t' - x / c$. In the boosted frame, the spatial dimensions along the y -axis undergo length contraction; however, since the foil is considered infinite in the y -direction, the electron density transforms as $n' = n / \cos \theta$. Notably, the plasma frequency $\omega'_p = \omega_p$ remains invariant under this specific transformation. Furthermore, the ions and electrons acquire a longitudinal velocity $-V$ along the y' -direction. Consequently the ion momentum and energy in this reference frame are defined by: $\vec{p}'_i = -m_i c \tan \theta \hat{e}_y$ and $E'_i = m_i c^2 / \cos \theta$ respectively, where m_i is the mass of the ions and $m_i / m = 1836$. This streaming motion is central to the "sliding mirror" effect, as it dictates the initial momentum state of the plasma species before interaction with the laser pulse.

III.SLIDING MIRROR: HHG

The vector potential $\vec{A}'(x, t')$ that satisfies the following equation describes the way the laser pulse interacts with the thin plasma foil (placed at $x=0$) in the boosted frame [6], [8]

$$d\vec{A}'(0, t') / dt' - 2 \pi \vec{J}'(\vec{A}'(0, t')) = d\vec{A}'_0(0, t') / dt',$$

(3) where $\vec{A}'_0(0, t')$ denotes the vector potential of the incident pulse, and $\vec{J}'(\vec{A}'(0, t'))$ stands for the electric current. Here vector potential is normalized to mc^2 / e and time is normalized to ω_0^{-1} . Following the plane wave approximation for the electric current term one can get the following expression [6], [8]

$$\vec{J}'(\vec{A}'(0,t')) = \frac{\varepsilon_p}{2\pi \cos\theta} \left[\frac{\vec{p}'_0}{\sqrt{1+|\vec{p}'_0|^2}} - \frac{\vec{p}'_0 + \vec{A}'(0,t')}{\sqrt{1+|\vec{p}'_0 + \vec{A}'(0,t')|^2}} \right],$$

(4) here \vec{p}'_0 is initial momentum of the electron normalized to mc . Substituting Eq. (4) in Eq.

(3) one can obtain the equation

$$\frac{d\vec{A}'(0,t')}{dt'} = \frac{\varepsilon_p}{\cos\theta} \left[\frac{\vec{p}'_0}{\sqrt{1+|\vec{p}'_0|^2}} - \frac{\vec{p}'_0 + \vec{A}'(0,t')}{\sqrt{1+|\vec{p}'_0 + \vec{A}'(0,t')|^2}} \right] - \vec{E}'_0(0,t').$$

(5)

Here $\frac{d\vec{A}'_0(0,t')}{dt'} = -\vec{E}'_0(0,t')$ is the electric field of the incident pulse and acts as a driving force. The first term in right hand side is responsible for high order harmonic generation.

The energy E_i of the accelerated ions can be estimated by the expression

$E_i = (m_i^2 c^4 + p_i^2 c^2)^{1/2} - m_i c^2$. This can be expressed in terms of the incident laser flux

density $w(\psi)$ (laser fluence) defined as [2] $w(\psi) = \int_{-\infty}^{\psi} \frac{E_0^2(t)}{4\pi\omega_0 n l m_i c} dt$. Here

$\psi = \omega_0(t - x(t)/c)$ is the phase of the incident laser pulse.

IV. RESULT AND DISCUSSIONS

Numerical results were performed using MATLAB software to investigate the interaction dynamics. Fig. 1 illustrates the evolution of the vector potential of the reflected pulse as a function of the incident laser frequency. This relationship is analyzed across varying plasma density parameters, specifically $\varepsilon_p = 10, 20, 30$ and 40 and $\theta = \pi/3$. The data indicate a significant inverse relationship; the normalized vector potential undergoes a sharp attenuation as the plasma density parameter increases.

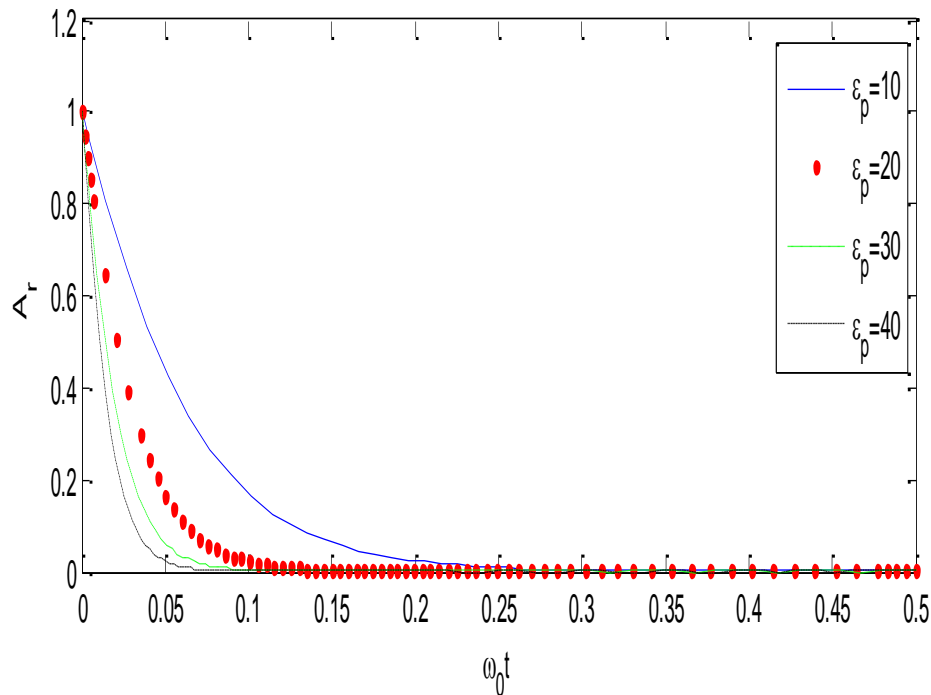


Fig. 1 illustrates the variation of the normalized amplitude of the reflected laser pulse with normalized frequency of the incoming laser pulse.

To model the driving force, a Gaussian intensity profile $E_0(0,t) = E_0 \exp(-t^2)$, was utilized for the laser pulse. Fig. 2 depicts the normalized energy of the accelerated ions relative to the parameter $\omega_0 t$. The results demonstrate that ion energy scales positively with $\omega_0 t$ before reaching a saturation point, thereafter maintaining a steady-state value. The reflective properties of the plasma foil were further quantified by examining the reflection coefficient R , which is the ratio of the time-averaged energy flux from the thin plasma surface to the incident flux. Following the analytical framework established in [8], the reflection coefficient for an s-polarized wave in the boosted frame is expressed as:

$$R = \left| \frac{\epsilon_p}{i \cos \theta + \epsilon_p} \right|^2$$
. Fig. 3 presents the variation of reflection coefficient with plasma density parameter ϵ_p for various angles of incidence θ . The numerical results reveal that for a constant plasma density, the reflection coefficient is monotonically enhanced by increasing the angle of incidence.

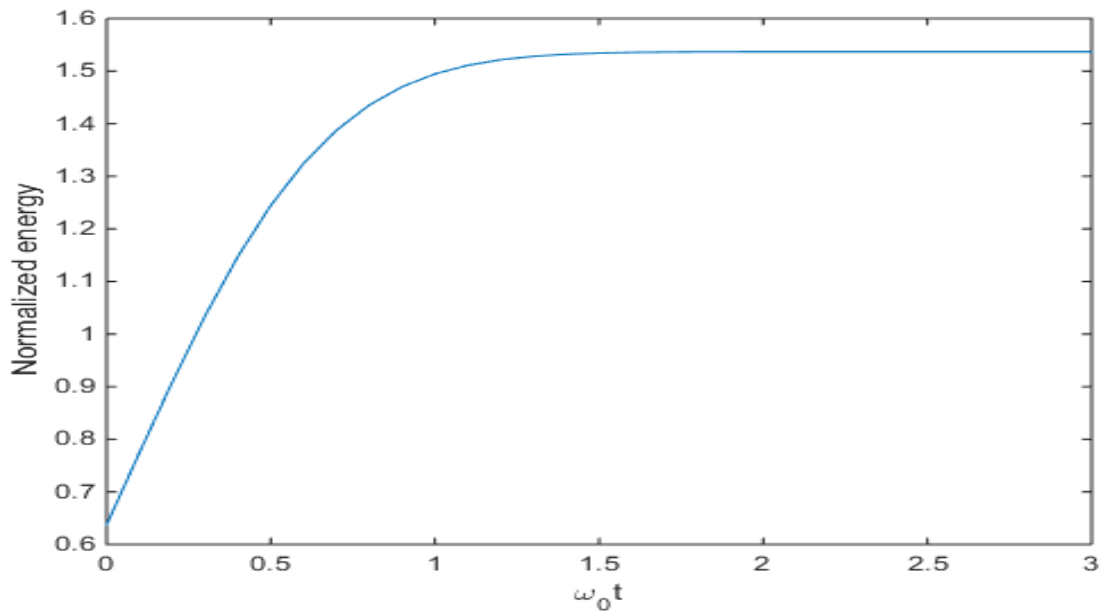


Fig. 2 depicts the variation of the normalized energy of the ions with the frequency of the electromagnetic pulse.

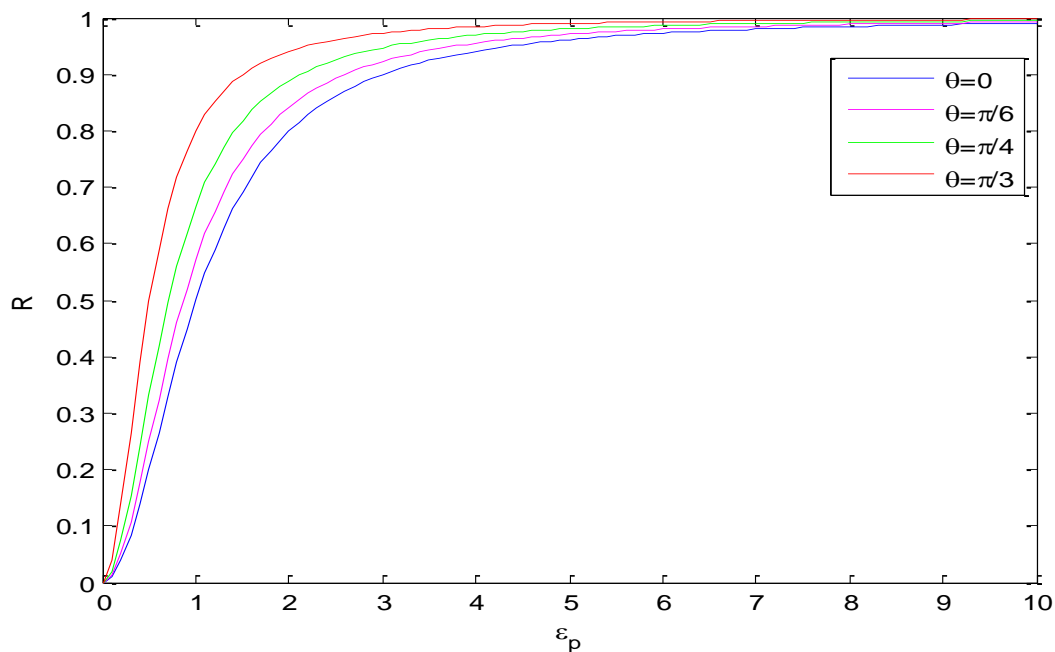


Fig. 3 shows the variation of the reflection coefficient R with plasma density parameter ϵ_p .

V. CONCLUSIONS



This work presents an analytical and numerical study of the nonlinear interaction of an obliquely incident electromagnetic wave with a relativistic sliding mirror. Our analysis identifies the transverse electronic current density, represented by the source term in Eq. (5), as the primary driver for HHG. Parametric studies reveal that the reflected wave amplitude exhibits an inverse relationship with the plasma density parameter; specifically, as the density parameter increases, a corresponding attenuation in the reflected signal is observed. Furthermore, for a static plasma density, the reflection coefficient scales positively with the angle of incidence, suggesting enhanced coupling at shallower angles. Finally, the kinetic energy of the accelerated ions is found to increase monotonically with the frequency, indicating a more efficient energy transfer in high-intensity regimes.

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